

Grundlagen der Teilchendetektoren

Werner Riegler, Sommersemester 2008

- ◆ **Geschichte der Instrumentierung – Teilchenphysik**
- ◆ **Grundlagen - ‘die Welt der Elementarteilchen’**
- ◆ **ElektroMagnetische Wechselwirkung geladener Teilchen mit Materie**
- ◆ **Detektoren basierend auf Nachweis angeregter Zustände**
- ◆ **Detektoren basierend auf Nachweis von Ionisationsladung, Spurdetektoren (Gasdetektoren, Festkörperdetektoren)**

Particle Detectors

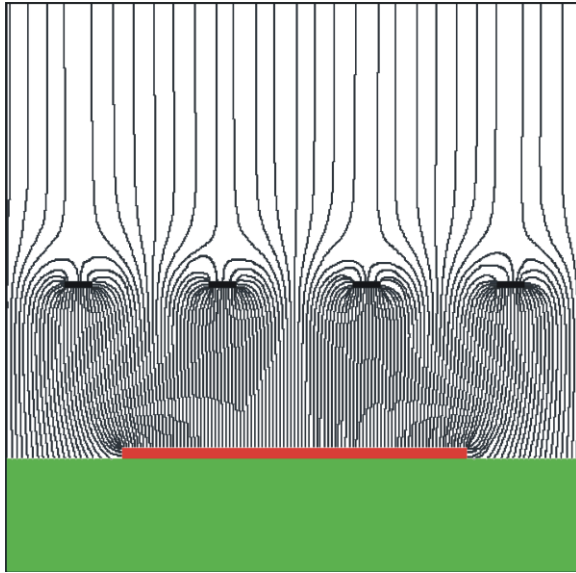
Detector-Physics: Precise knowledge of the processes leading to signals in particle detectors is necessary.

The detectors are nowadays working close to the limits of theoretically achievable measurement accuracy – even in large systems.

Due to available computing power, detectors can be simulated to within 5-10% of reality, based on the fundamental microphysics processes (atomic and nuclear crosssections).

Particle Detector Simulation

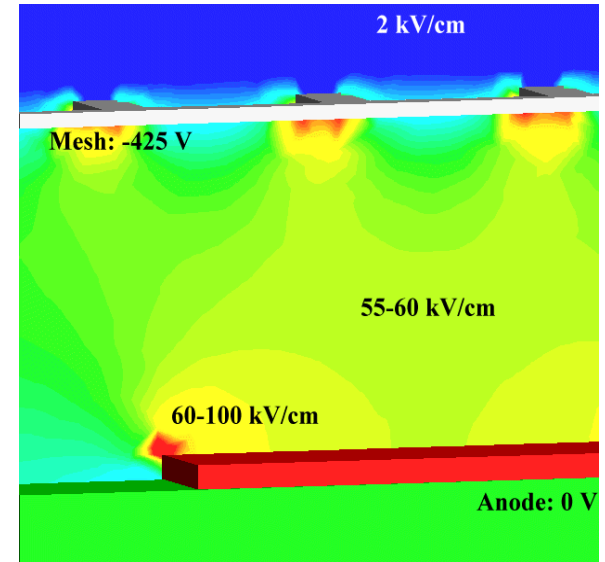
Electric Fields in a Micromega Detector



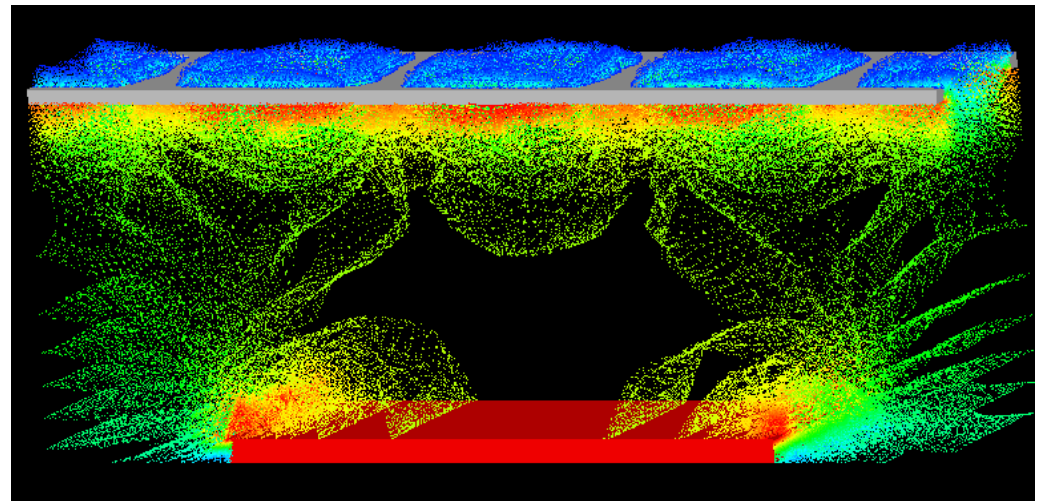
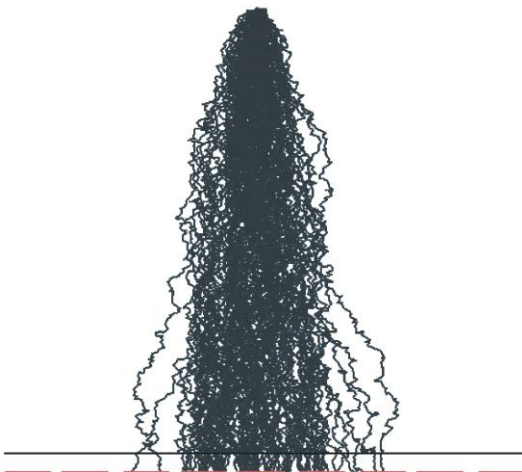
Very accurate simulations of particle detectors are possible due to availability of Finite Element simulation programs and computing power.

Follow every single electron by applying first principle laws of physics.

Electric Fields in a Micromega Detector



Electrons avalanche multiplication

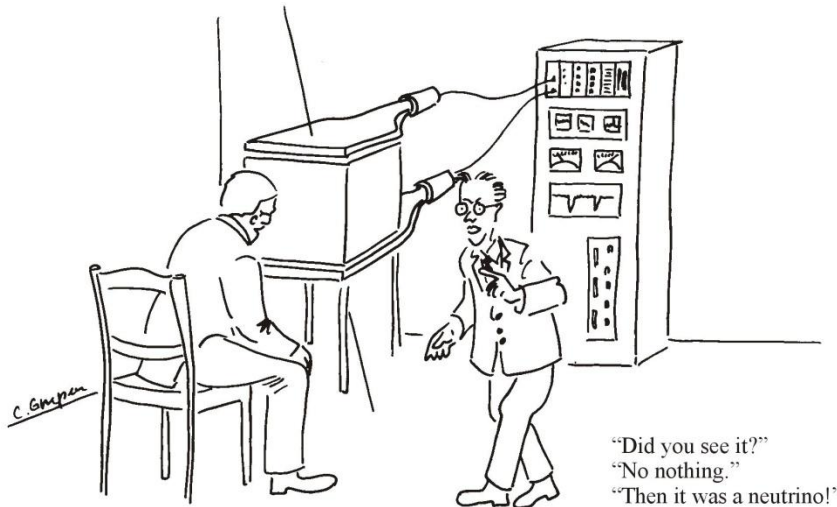


Interaction of Particles with Matter

Any device that is to detect a particle must interact with it in some way → almost ...

In many experiments neutrinos are measured by missing transverse momentum.

E.g. e^+e^- collider. $P_{\text{tot}}=0$,
If the Σp_i of all collision products is $\neq 0$ → neutrino escaped.



Electro-Magnetic Interaction of Particles with Matter

Various aspects of the penetration of charged particles in matter have occupied the thoughts of some of the finest physicists of the last century.

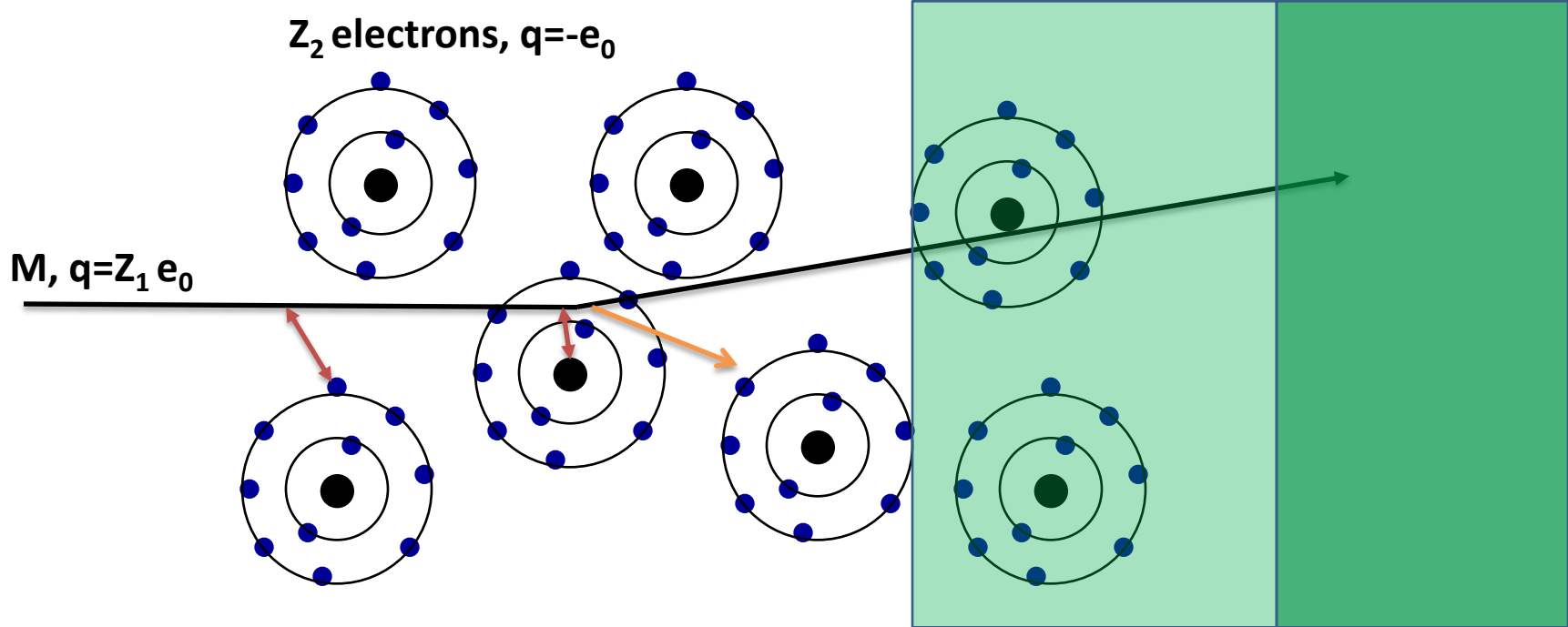
E.g. Thomson 1903, Rutherford 1911, Bohr 1913, 1915, 1948, Bethe 1930, 1932, Mott, 1931, Bloch 1933, Fermi 1940, Landau 1944

In the first half of the 20th century, the energy loss of the charged particles and the related stopping power of materials was the prime issue.

Nowadays, the actual amount of scintillation light and/or charge produced by the passing particle, and the fluctuations of these quantities, are the important quantity because these are the quantities produce the signals in particle detectors and their fluctuations are responsible for the resolution limits of the detectors.

We will therefore summarize the basic mechanisms that are responsible for the creation of excitation and ionization and will explain the models that are implemented in modern simulation programs like GEANT and HEED.

Electromagnetic Interaction of Particles with Matter

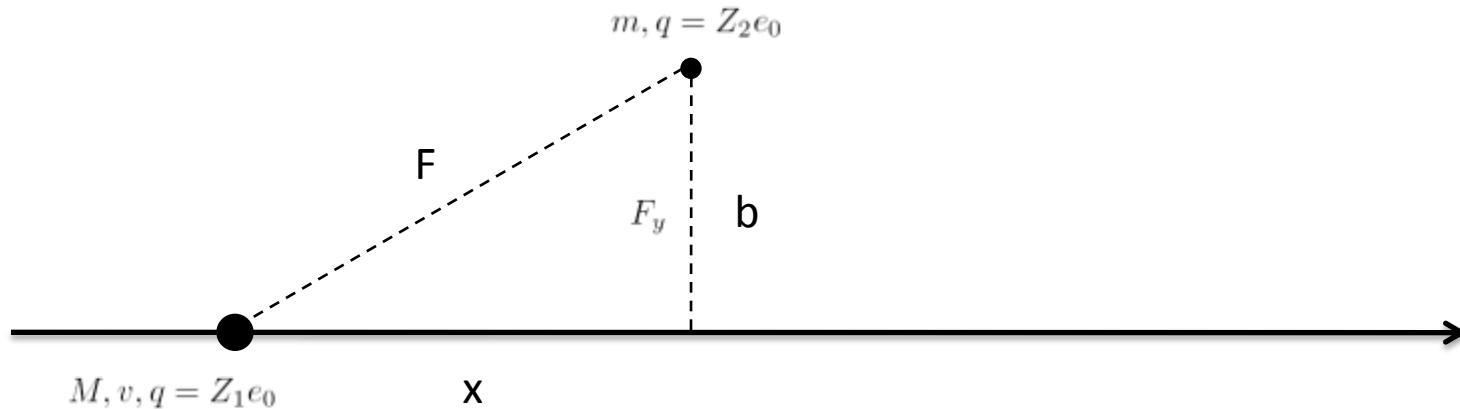


Interaction with the atomic electrons. The incoming particle loses energy and the atoms are excited or ionized.

Interaction with the atomic nucleus. The particle is deflected (scattered) causing multiple scattering of the particle in the material. During this scattering a Bremsstrahlung photon can be emitted.

In case the particle's velocity is larger than the velocity of light in the medium, the resulting EM shockwave manifests itself as Cherenkov Radiation. When the particle crosses the boundary between two media, there is a probability of the order of 1% to produce and X ray photon, called Transition radiation.

Interaction of Particles with Matter



While the charged particle is passing another charged particle, the Coulomb Force is acting, resulting in momentum transfer

$$F_y = \frac{Z_1 Z_2 e_0^2}{4\pi\epsilon_0(b^2 + v^2 t^2)} \frac{b}{\sqrt{b^2 + v^2 t^2}} \quad \Delta p = \int_{-\infty}^{\infty} F_y(t) dt = \frac{2Z_1 Z_2 e_0^2}{4\pi\epsilon_0 v b}$$

The relativistic form of the transverse electric field doesn't change the momentum transfer. The transverse field is stronger, but the time of action is shorter

$$F_y = \frac{\gamma Z_1 Z_2 e_0^2 b}{4\pi\epsilon_0(b^2 + \gamma^2 v^2 t^2)^{3/2}} \quad \Delta p = \int_{-\infty}^{\infty} F_y(t) dt = \frac{2Z_1 Z_2 e_0^2}{4\pi\epsilon_0 v b}$$

The transferred energy is then

$$\Delta E = \frac{(\Delta p)^2}{2m} = \frac{Z_2^2}{m} \frac{2Z_1^2 e_0^4}{(4\pi\epsilon_0)^2 v^2 b^2}$$

$$\Delta E(\text{electrons}) = Z_2 \frac{1}{m_e} \frac{2Z_1^2 e_0^4}{(4\pi\epsilon_0)^2 v^2 b^2} \quad \Delta E(\text{nucleus}) = \frac{Z_2^2}{2Z_2 m_p} \frac{2Z_1^2 e_0^4}{(4\pi\epsilon_0)^2 v^2 b^2} \quad \frac{\Delta E(\text{electrons})}{\Delta E(\text{nucleus})} = \frac{2m_p}{m_e} \approx 4000$$

→ The incoming particle transfer energy only to the atomic electrons !

Interaction of Particles with Matter

Target material: mass A , Z_2 , density ρ [g/cm³], Avogadro number N_A

A gramm $\rightarrow N_A$ Atoms:

Number of atoms/cm³

$n_a = N_A \rho / A$ [1/cm³]

Number of electrons/cm³

$n_e = N_A \rho Z_2 / A$ [1/cm³]

$$\Delta E(\text{electrons}) = \frac{2Z_2 Z_1^2 m_e c^2}{\beta^2 b^2} \frac{e_0^4}{(4\pi\epsilon_0 m_e c^2)^2} = \frac{2Z_2 Z_1^2 m_e c^2}{\beta^2 b^2} r_e^2$$



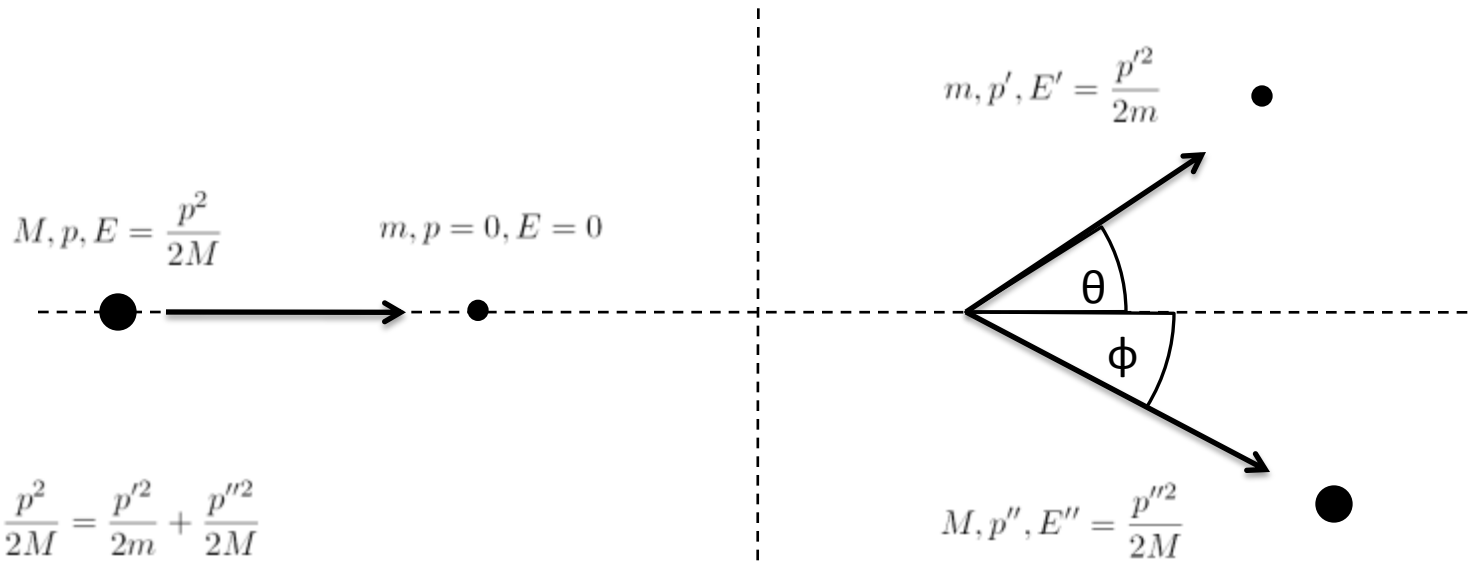
$$dE = - \int_{b_{min}}^{b_{max}} n_e \Delta E dx 2\pi b db = - \frac{4\pi Z_2 Z_1^2 m_e c^2 r_e^2}{\beta^2} \frac{N_A \rho}{A} \int_{b_{min}}^{b_{max}} \frac{db}{b}$$

With $\Delta E(b) \rightarrow db/b = -1/2 dE/E \rightarrow E_{max} = \Delta E(b_{min})$ $E_{min} = \Delta E(b_{max})$

$$\frac{dE}{dx} = -2\pi r_e^2 m_e^2 c^2 \frac{Z_1^2}{\beta^2} \frac{N_A Z_2 \rho}{A} \int_{E_{min}}^{E_{max}} \frac{dE}{E} = -2\pi r_e^2 m_e^2 c^2 \frac{Z_1^2}{\beta^2} \frac{N_A Z_2 \rho}{A} \ln \frac{E_{max}}{E_{min}}$$

$E_{min} \approx I$ (Ionization Energy)

Nonrelativistic Collision Kinematics, E_{max}



$$1) \quad \frac{p^2}{2M} = \frac{p'^2}{2m} + \frac{p''^2}{2M}$$

$$2) \quad \begin{aligned} p &= p' \cos \theta + p'' \cos \phi & p''^2 &= p'^2 + p^2 - 2pp' \cos \theta \\ 0 &= p' \sin \theta + p'' \sin \phi \end{aligned}$$

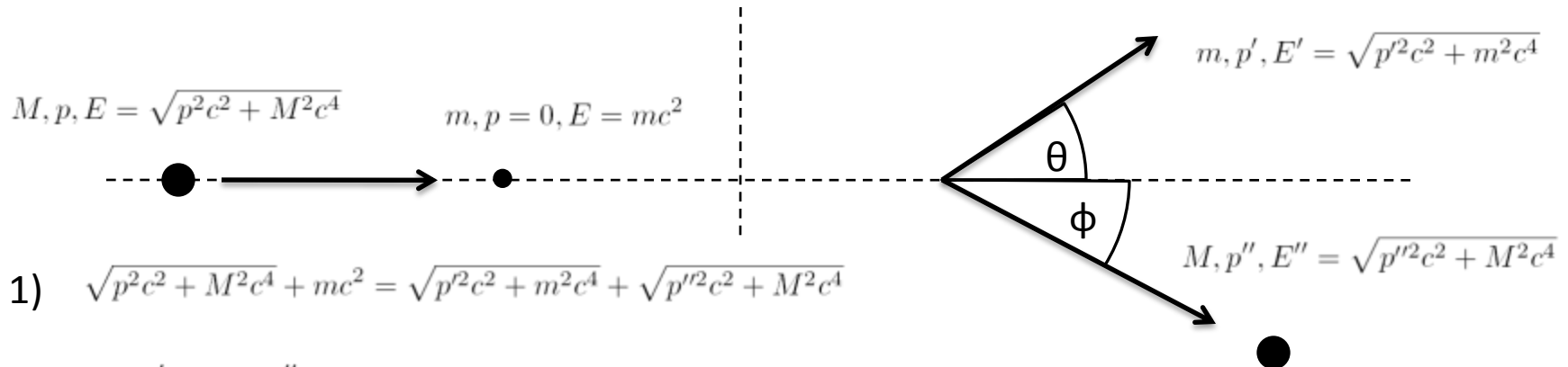
$$1+2) \quad p' = \frac{2mp \cos \theta}{M + m} \quad \rightarrow \quad E' = \frac{p'^2}{2m} = \frac{2mp^2 \cos^2 \theta}{(m + M)^2}$$

$$M = m \quad \rightarrow \quad \frac{E'_{max}}{E} = 1$$

$$E'_{max} = \frac{2mp^2}{(m + M)^2} \quad \rightarrow \quad \frac{E'_{max}}{E} = \frac{4mM}{(m + M)^2} \leq 1$$

$$M = 200m \quad \rightarrow \quad \frac{E'_{max}}{E} = 0.02$$

Relativistic Collision Kinematics, E_{\max}



$$1) \quad \sqrt{p^2 c^2 + M^2 c^4} + mc^2 = \sqrt{p'^2 c^2 + m^2 c^4} + \sqrt{p''^2 c^2 + M^2 c^4}$$

$$2) \quad p = p' \cos \theta + p'' \cos \phi$$

$$0 = p' \sin \theta + p'' \sin \phi$$

$$p''^2 = p'^2 + p^2 - 2pp' \cos \theta$$

$$1+2) \quad E^{k'} = \sqrt{p'^2 c^2 + m^2 c^4} - mc^2 = \frac{2mc^2 p^2 c^2 \cos^2 \theta}{\left[mc^2 + \sqrt{p^2 c^2 + M^2 c^4} \right]^2 - p^2 c^2 \cos^2 \theta}$$

$$E^{k' \max} = \frac{2mc^2 p^2 c^2}{(m^2 + M^2)c^4 + 2m\sqrt{p^2 c^2 + M^2 c^4}} = 2mc^2 \beta^2 \gamma^2 F \quad F = \left(1 + \frac{2m}{M} \sqrt{1 + \beta^2 \gamma^2} + \frac{m^2}{M^2} \right)^{-1}$$

$$\frac{E^{k' \max}}{E^k} = \frac{E^{k' \max}}{\sqrt{p^2 c^2 + M^2 c^4} - Mc^2} = \frac{2mc^2 p^2 c^2}{Mc^2 \left(\sqrt{1 + \frac{p^2 c^2}{M^2 c^4}} - 1 \right) \left(m^2 c^4 + M^2 c^4 + 2mM c^4 \sqrt{1 + \frac{p^2 c^2}{M^2 c^4}} \right)}$$

$$pc \ll Mc^2 \quad \rightarrow \quad \frac{E^{k' \max}}{E^k} = \frac{4mM}{(m+M)^2} \quad pc \gg Mc^2 \quad \rightarrow \quad \frac{E^{k' \max}}{E^k} = 1$$

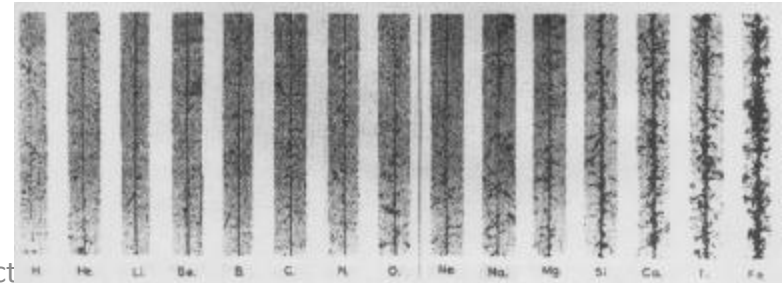
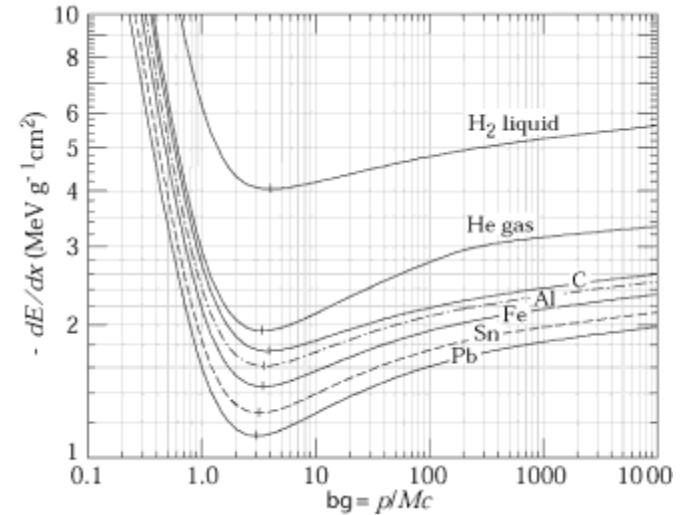
Specific Energy Loss

$$\frac{1}{\rho} \frac{dE}{dx} = -2\pi r_e^2 m_e c^2 N_A \frac{Z_1^2}{\beta^2} \frac{Z_2}{A} \ln \frac{2m_e c^2 \beta^2 \gamma^2 F}{I} \quad F = \left(1 + \frac{2m_e}{M} \sqrt{1 + \beta^2 \gamma^2} + \frac{m_e^2}{M^2} \right)^{-1}$$

This formula is up to a factor two identical with the correct QM formula.

The specific Energy Loss $1/\rho \, dE/dx$

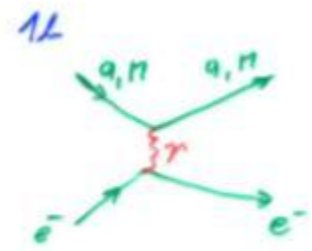
- decreases at $1/\beta^2$ of the incoming particle
- increases like $\ln \gamma$ for $\beta = 1$
- is \approx independent of M ($M \gg m_e$)
- is proportional to Z_1^2 of the incoming particle
- is \approx independent of the Material ($Z/A \approx \text{const.}$)
- $dE/dx \approx 1-2 \times \rho \text{ [g/cm}^3\text{]} \text{ MeV/cm}$



Crosssection

Crosssection σ : Material with Atomic Mass A and density ρ contains n Atoms/cm³

$$n[\text{cm}^{-3}] = \frac{N_A[\text{mol}^{-1}] \rho[\text{g}/\text{cm}^3]}{A[\text{g}/\text{mol}]} \quad N_A = 6.022 \times 10^{23} \text{ mol}^{-1}$$



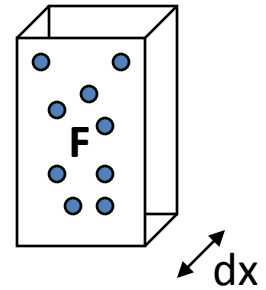
E.g. Atom (Sphere) with Radius R : Atomic Crosssection $\sigma = R^2\pi$

A volume with surface F and thickness dx contains $N=nFdx$ Atoms.

The total 'surface' of atoms in this volume is $N \sigma$.

The relative area is $p = N \sigma / F = N_A \rho \sigma / A dx =$

Probability that an incoming particle hits an atom in dx .



What is the probability P that a particle hits an atom between distance x and $x+dx$?

P = probability that the particle does NOT hit an atom in the $m=x/dx$ material layers and that the particle DOES hit an atom in the m^{th} layer

$$P(x)dx = (1-p)^m p \approx e^{-m} p = \exp\left(-\frac{N_A \rho \sigma}{A} x\right) \frac{N_A \rho \sigma}{A} dx = \frac{1}{\lambda} \exp\left(-\frac{x}{\lambda}\right) dx \quad \lambda = \frac{A}{N_A \rho \sigma}$$

Mean free path $= \int_0^{\infty} x P(x) dx = \int_0^{\infty} \frac{x}{\lambda} e^{-\frac{x}{\lambda}} dx = \lambda$

Average number of collisions/cm $= \frac{1}{\lambda} = \frac{N_A \rho \sigma}{A}$

Differential Crosssection



Differential Crosssection: $\frac{d\sigma(E, E')}{dE'}$

→ Crosssection for an incoming particle of energy E to lose an energy between E' and E'+dE'

Total Crosssection: $\sigma(E) = \int \frac{d\sigma(E, E')}{dE'} dE'$

Probability P(E) that an incoming particle of Energy E loses an energy between E' and E'+dE' in a collision:

$$P(E, E')dE' = \frac{1}{\sigma(E)} \frac{d\sigma(E, E')}{dE'} dE'$$

Average number of collisions/cm causing an energy loss between E' and E'+dE' $= \frac{N_A \rho}{A} \frac{d\sigma(E, E')}{dE'}$

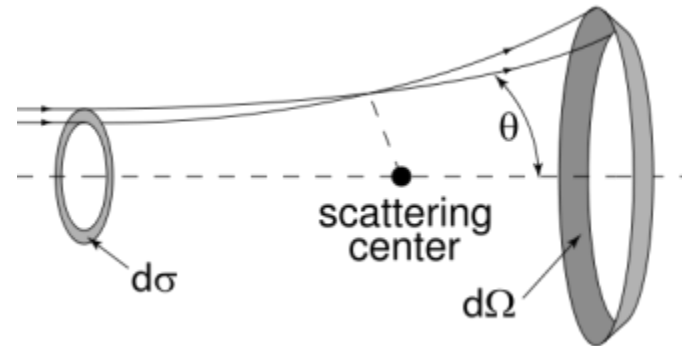
Average energy loss/cm: $\frac{dE}{dx} = -\frac{N_A \rho}{A} \int E' \frac{d\sigma(E, E')}{dE'} dE'$

Rutherford Scattering

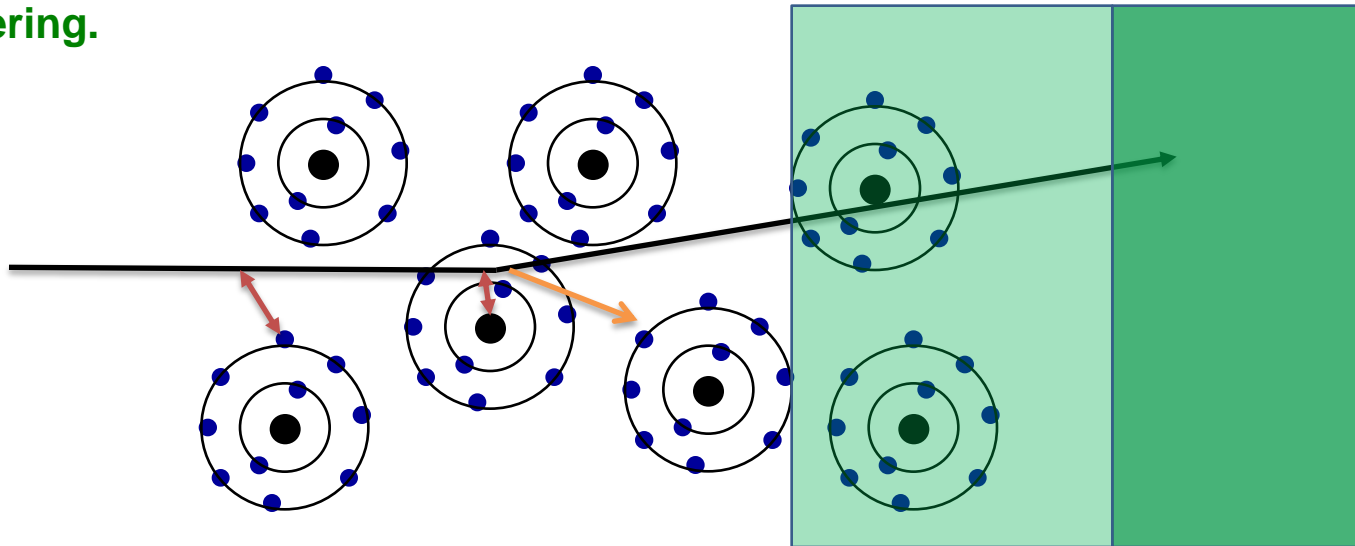
A particle of mass M and charge Z_1e_0 is scattering off a scattering center of charge Z_2e_0 by interaction through the Coulomb force.

The differential cross section for finding the particle scattered into a solid angle $d\Omega$ is given by ($p=\gamma\beta Mc$ and $v = \beta c$).

$$\frac{d\sigma}{d\Omega} = \left(\frac{1}{4\pi\epsilon_0} \frac{Z_1Z_2e_0^2}{2pv} \right)^2 \frac{1}{\sin^4 \theta/2}$$



The incoming particle is typically much heavier than the atomic electrons \rightarrow excitation and ionization, and much lighter than the nucleus \rightarrow Multiple scattering.



Rutherford Scattering off Electrons

Jackson, 13.1: Assuming that the incoming particle is much heavier than the atomic electrons ($M \gg m_e$), the collision is best viewed as elastic Coulomb scattering in the rest frame of the incident particle. Expressing the scattering angle in terms of energy transfer E' of the incoming particle to the electron we find

$$\frac{d\sigma}{dE'} = \left(\frac{1}{4\pi\epsilon_0} \right)^2 \frac{2\pi Z_1^2 e_0^4}{m_e c^2 \beta^2} \frac{1}{E'^2} = Z_1^2 r_e^2 \frac{2\pi m_e c^2}{\beta^2} \frac{1}{E'^2}$$

The energy loss is therefore

$$\frac{dE}{dx} = -\frac{N_A Z_2 \rho}{A} \int E' \frac{d\sigma}{dE'} dE' = -2\pi r_e^2 m_e c^2 \frac{Z_1^2 N_A Z_2 \rho}{\beta^2 A} \ln \frac{E_{max}}{E_{min}}$$

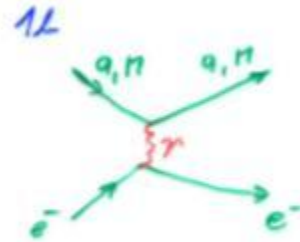
This is exactly the same result that we found before. In order to shed light on the question of atomic binding we have to treat the problem with QM.

QM Treatment of Energy Loss

For large energy transfers $E' > \eta \sim 50\text{keV}$ the electrons can be considered quasi free and the relativistic formula for the scattering of a spin 0 particle of mass M and Charge $Z_1 e_0$ on a free electron can be used (Bhabha 1938)

$$\frac{d\sigma(E, E')}{dE'} = \frac{2\pi Z_1^2 r_e^2 m_e c^2}{\beta^2} \frac{1}{E'^2} \left(1 - \beta^2 \frac{E'}{E_{max}} \right)$$

For small energy transfers $E' \ll E_{max}$ this is equal to the Rutherford crosssection. For large energy transfers i.e. very small impact parameters (close encounters), the electron spin gives rise to a correction.

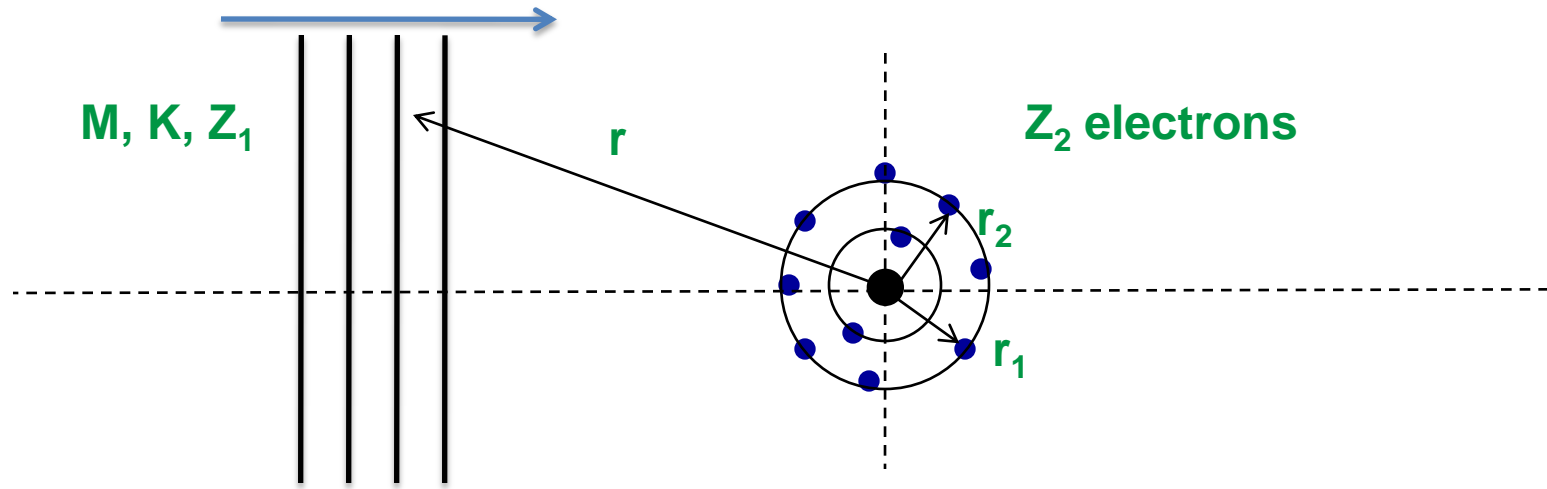


The energy loss due to these high energy collisions is then

$$\frac{dE}{dx} |_{E' > \eta} = -\frac{N_A Z_2 \rho}{A} \int_{\eta}^{E_{max}} \frac{d\sigma}{dE'} dE' = -2\pi r_e^2 m_e c^2 \frac{Z_1^2}{\beta^2} \frac{N_A Z_2 \rho}{A} \left(\ln \frac{E_{max}}{\eta} - \beta^2 \right)$$

For small energy transfers E' the atomic excitations have to be taken into account. Bethe 1930, 1932 gave a fully QM treatment of the problem. The main steps of the derivation are outlined in the following:

QM Treatment of Energy Loss, Bethe 1930



The QM system consists of the incoming particle and the atom. The interaction between the two is defined by V_{int}

$$H = H_{Atom}^0 + H_{Part}^0 + V_{int} \quad i\hbar \frac{d}{dt} \psi = H\psi$$

$$H_{Atom}^0 \psi_n(\vec{r}_1, \vec{r}_2, \dots, \vec{r}_{Z_2}) = E_n \psi_n(\vec{r}_1, \vec{r}_2, \dots, \vec{r}_{Z_2}) \quad H_{Part}^0 \Phi_K(\vec{r}) = \frac{\hbar^2 K^2}{2M} \Phi_K(\vec{r}) \quad \Phi_K(\vec{r}) = e^{i\vec{K}\vec{r}}$$

$$V_{int} = \frac{Z_2}{r} - \sum_{i=1}^{Z_2} \frac{1}{|\vec{r} - \vec{r}_i|}$$

QM Treatment of Energy Loss

The probability, and associated crosssection, that an incoming particle with momentum $\hbar\mathbf{K}$ excites the atom to state n and is leaving with momentum $\hbar\mathbf{K}'$ can be calculated to first order by applying Born's approximation.

$$u_i = e^{i\vec{K}\vec{r}}\psi_0(\vec{r}_i)e^{-i/\hbar W t} \quad W = \frac{\hbar^2 K^2}{2M} + E_0 \quad u_o = e^{i\vec{K}'\vec{r}}\psi_n(\vec{r}_i)e^{-i/\hbar W' t} \quad W' = \frac{\hbar^2 K'^2}{2M} + E_n$$

$$V_{0n}(K, K') = \langle u_o | V_{int} | u_i \rangle = \int \left(\frac{Z_2}{r} - \sum_{i=1}^{Z_2} \frac{1}{|\vec{r} - \vec{r}_i|} \right) \psi_0(\vec{r}_i) \bar{\psi}_n(r_i) e^{i(\vec{K} - \vec{K}')\vec{r}} d^3r d^3r_1 \dots d^3r_{Z_2}$$

$$\frac{d\sigma_n(K, K')}{d\theta} = \frac{\alpha^2}{2\pi} \left(\frac{M}{m} Z_1 \right)^2 |V_{0n}|^2 \frac{K'}{K} \sin\theta \quad \vec{q} = \vec{K} - \vec{K}' \quad q = |\vec{q}| \quad \Delta p = \hbar q$$

This expression can be transformed into

$$\frac{d\sigma_n(q)}{dq} = \frac{8\pi\alpha^2}{K^2} \left(\frac{M}{m} Z_1 \right)^2 \frac{1}{q^3} |\epsilon_n(q)|^2 \quad \epsilon_n(q) = \int \left(Z_2 - \sum_{j=1}^{Z_2} e^{i(\vec{q}\vec{r}_j)} \right) \psi_0(\vec{r}_j) \bar{\psi}_n(r_j) d^3r_1 \dots d^3r_{Z_2}$$

QM Treatment of Energy Loss

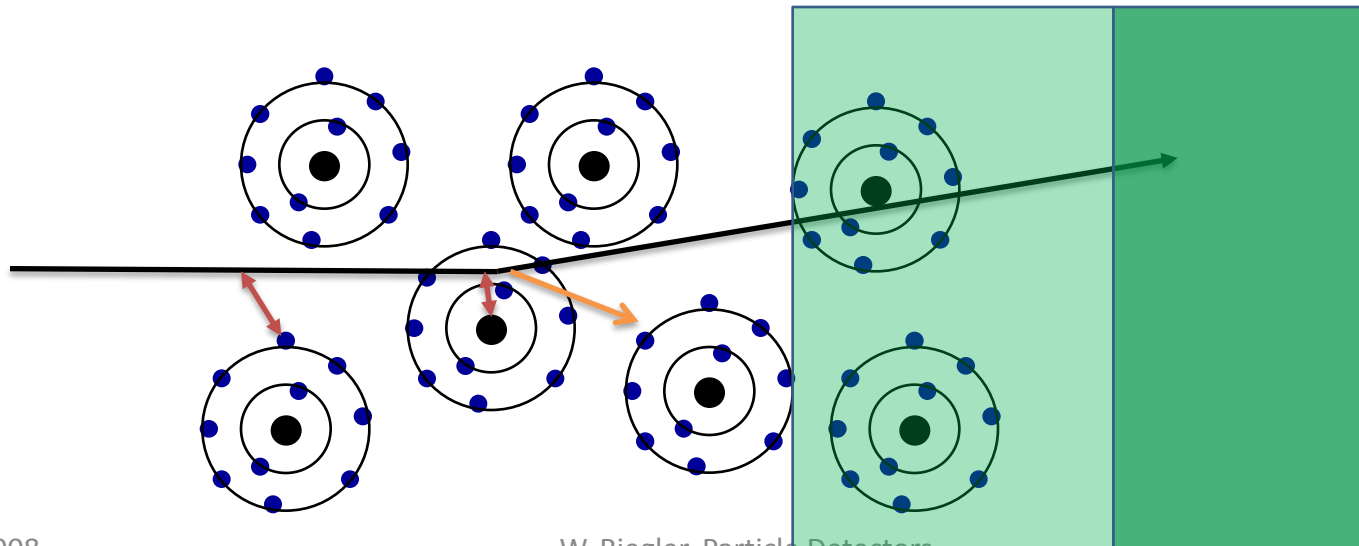
For elastic scattering, where the incoming particle doesn't lose energy, we set $n=0$ and have

$$\epsilon_0(q) = Z_2 - \sum_{j=1}^{Z_2} \int e^{i(\vec{q}\vec{r}_j)} \psi_0^2(\vec{r}_j) d^3r_1 \dots d^3r_{Z_2} = Z_2 - F$$

where F is the atomic form factor (dependent on q) from the theory of X-ray scattering. Expressing the elastic crosssection by the scattering angle we find

$$\frac{d\sigma}{d\Omega} = \left(\frac{1}{4\pi\epsilon_0} \frac{Z_1(Z_2 - F)e_0^2}{2pv} \right)^2 \frac{1}{\sin^4 \theta/2}$$

This is again the Rutherford crosssection for scattering off the Nucleus that is partially 'shielded' by the electrons. We will use it later for calculation of multiple scattering.



QM Treatment of Energy Loss

For inelastic scattering, where the incoming particle excites the Atom, we set $n > 0$ and have

$$\frac{d\sigma_n(q)}{dq} = \frac{8\pi\alpha^2}{K^2} \left(\frac{M}{m} Z_1\right)^2 \frac{1}{q^3} |\epsilon_n(q)|^2 \quad \epsilon_n(q) = - \sum_{j=1}^{Z_2} \int e^{i(\vec{q}\vec{r}_j)} \psi_0(\vec{r}_j) \bar{\psi}_n(\vec{r}_j) d^3r_1 \dots d^3r_{Z_2}$$

The (lengthy) evaluation of this expression for the nonrelativistic case (Bethe 1930) and the relativistic case (Bethe 1932), using exact expressions for the Hydrogen atom and sum rules for the higher Z atoms, leads to the energy loss assuming energy transfers $E' < \eta$

$$\frac{dE}{dx}|_{E' < \eta} = - \sum_n (E_n - E_0) \int \frac{d\sigma_n(q)}{dq} dq = \sum_n E'_n \int \frac{d\sigma_n(q)}{dq} dq \quad E'_n < \eta$$

$$\frac{dE}{dx}|_{E' < \eta} = -2\pi r_e^2 m_e c^2 \frac{Z_1^2}{\beta^2} \frac{N_A Z_2 \rho}{A} \left(\ln \frac{E_{max} \eta}{I^2} - \beta^2 \right)$$

Adding the expressions for $E' < \eta$ and $E' > \eta$ leads to the final expression for the Energy loss

$$\frac{dE}{dx} = \frac{dE}{dx}|_{E' < \eta} + \frac{dE}{dx}|_{E' > \eta} = -4\pi r_e^2 m_e c^2 \frac{Z_1^2}{\beta^2} \frac{N_A Z_2 \rho}{A} \left(\ln \frac{E_{max}}{I} - \beta^2 \right)$$

The entire complexity of the atomic physics ends up in the average ionization potential I .

Bethe Bloch Formula

$$\frac{1}{\rho} \frac{dE}{dx} = -4\pi r_e^2 m_e c^2 \frac{Z_1^2}{\beta^2} N_A \frac{Z}{A} \left[\ln \frac{2m_e c^2 \beta^2 \gamma^2 F}{I} - \beta^2 - \frac{\delta(\beta\gamma)}{2} \right]$$

Density effect

$$\delta(\beta\gamma) = \ln h\omega_p/I + \ln \beta\gamma - \frac{1}{2}$$

For very high momenta the polarization of the medium by the strong transverse field, which reduces the Energy loss, must be taken into account.

At large Energy transfer (delta electrons) the produced electron can leave the material. In reality, E_{\max} must therefore be replaced by T_{cut} and the energy loss reaches a plateau.

We distinguish three distinct regions of energy loss as a function of the particles momentum

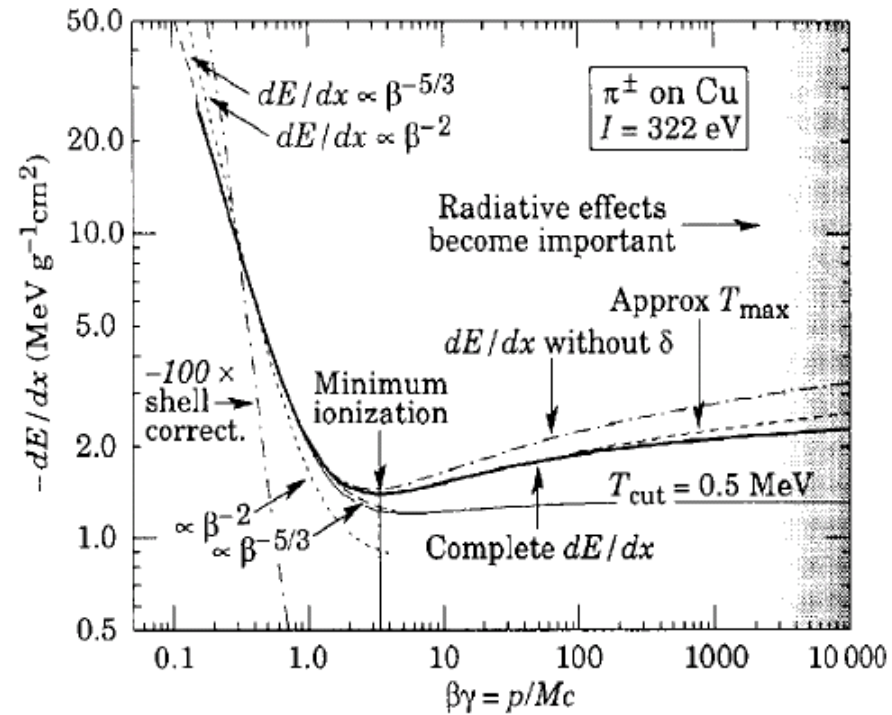
1) $1/\beta^2$ region with Minimum at $\beta\gamma \approx 3$

2) Relativistic Rise

3) Density Effect und Saturation

The Energy loss depends on the particle's velocity and is independent of it's Mass

For $Z > 1$, $I \approx 16Z^{0.9} \text{ eV}$



Bethe Bloch Formula

Für $Z \approx 0.5$ A

$1/\rho \, dE/dx \approx 1.4 \text{ MeV cm}^2/\text{g}$ für $\beta\gamma \approx 3$

Example 1:

Scintillator: Thickness = 2 cm; $\rho = 1.05 \text{ g/cm}^3$

Particle with $\beta\gamma = 3$ and $Z=1$

$1/\rho \, dE/dx \approx 1.4 \text{ MeV}$

$dE \approx 1.4 * 2 * 1.05 = 2.94 \text{ MeV}$

Example 2:

Iron: Thickness = 100 cm; $\rho = 7.87 \text{ g/cm}^3$

$dE \approx 1.4 * 100 * 7.87 = 1102 \text{ MeV} = 1.1 \text{ GeV}$

Example 3:

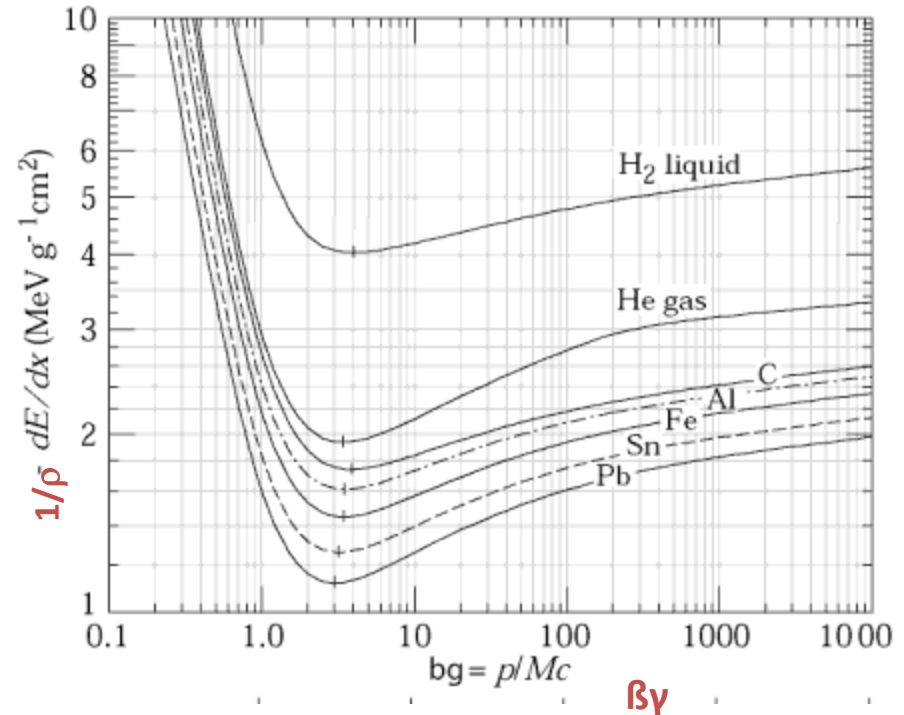
Energy Loss of Carbon – Ions with $Z=6$ and Momentum of 330 MeV/Nucleon

in Water, i.e. $\beta\gamma = p/m = 330/940 \approx .35 \rightarrow$

$\beta \approx .33$

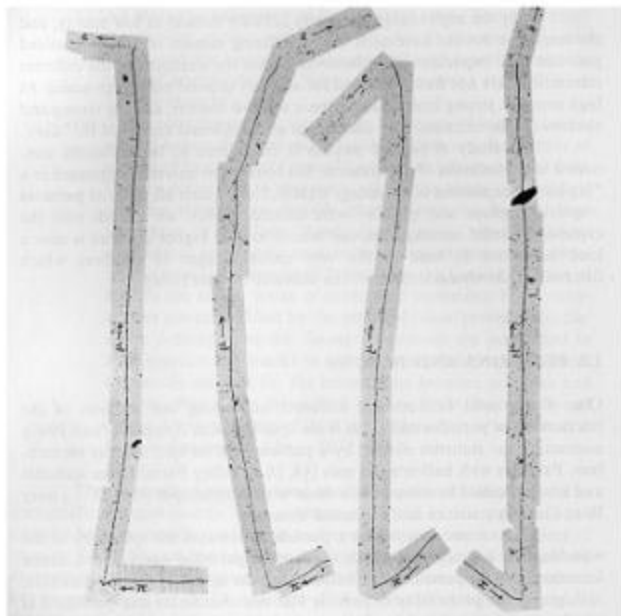
$dE/dx \approx 1.4 Z^2 / \beta^2 \approx 460 \text{ MeV/cm} \rightarrow$

Cancer Therapy !

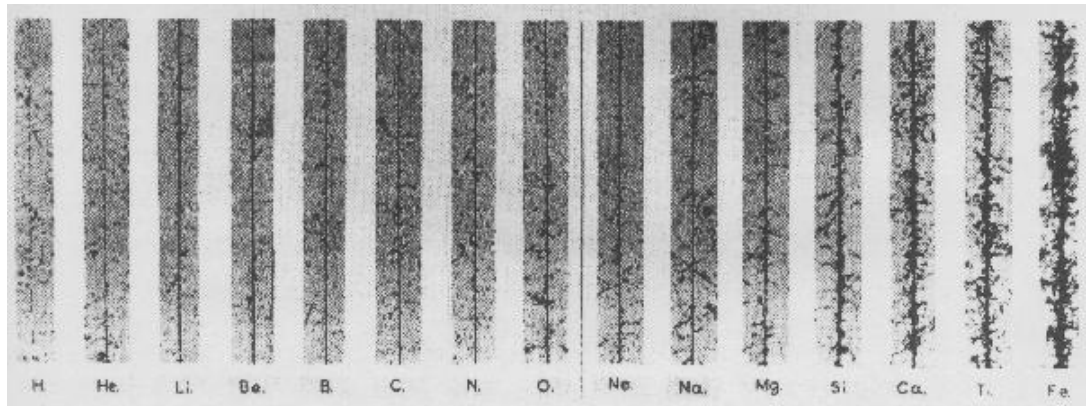


This number must be multiplied with the material density ρ [g/cm^3]
 $\rightarrow dE/dx$ [MeV/cm]

Small energy loss
→ Fast Particle

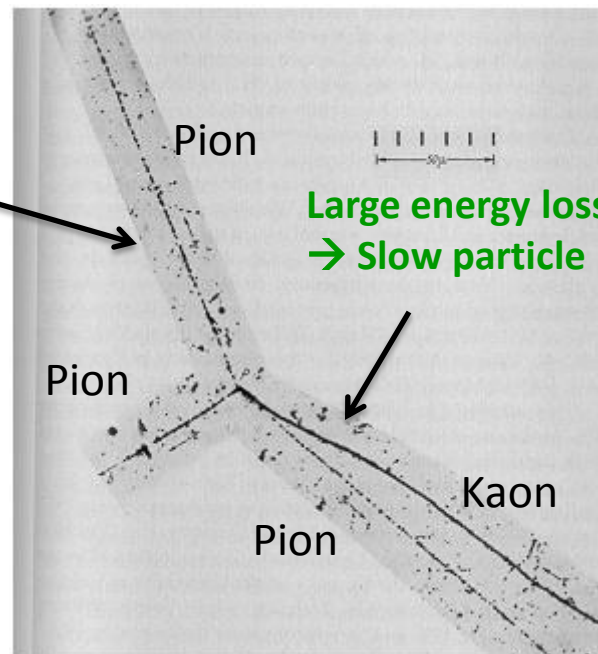


Discovery of muon and pion



Cosmic rays: $dE/dx \propto Z^2$

Small energy loss
→ Fast particle

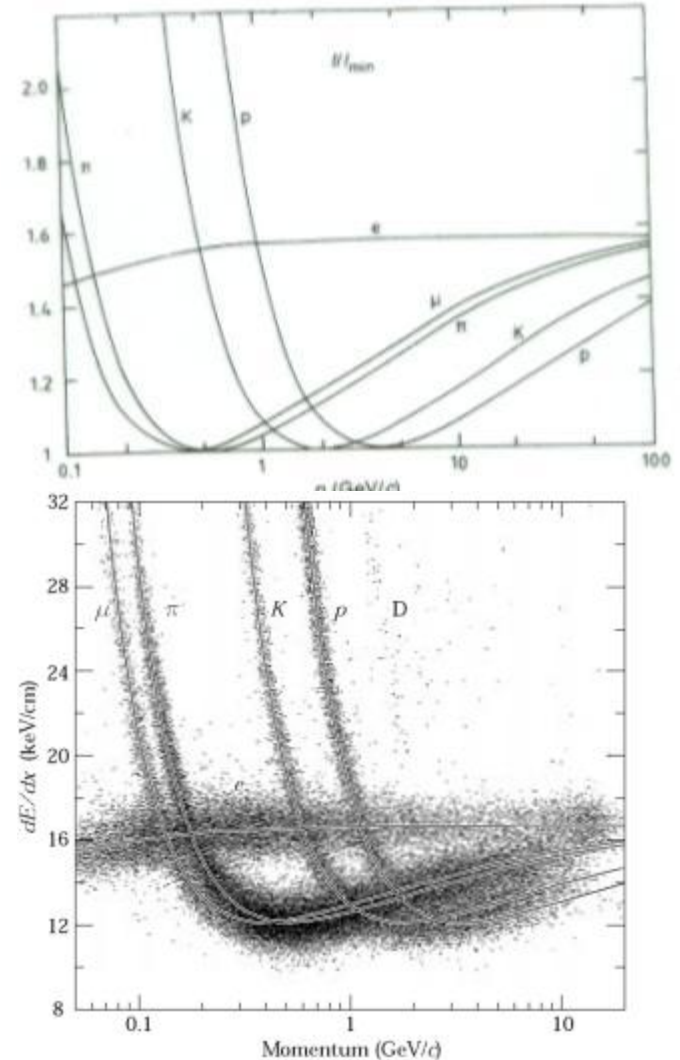


Bethe Bloch Formula, Particle ID

The energy loss as a function of the particle velocity is a universal function for particles of different masses.

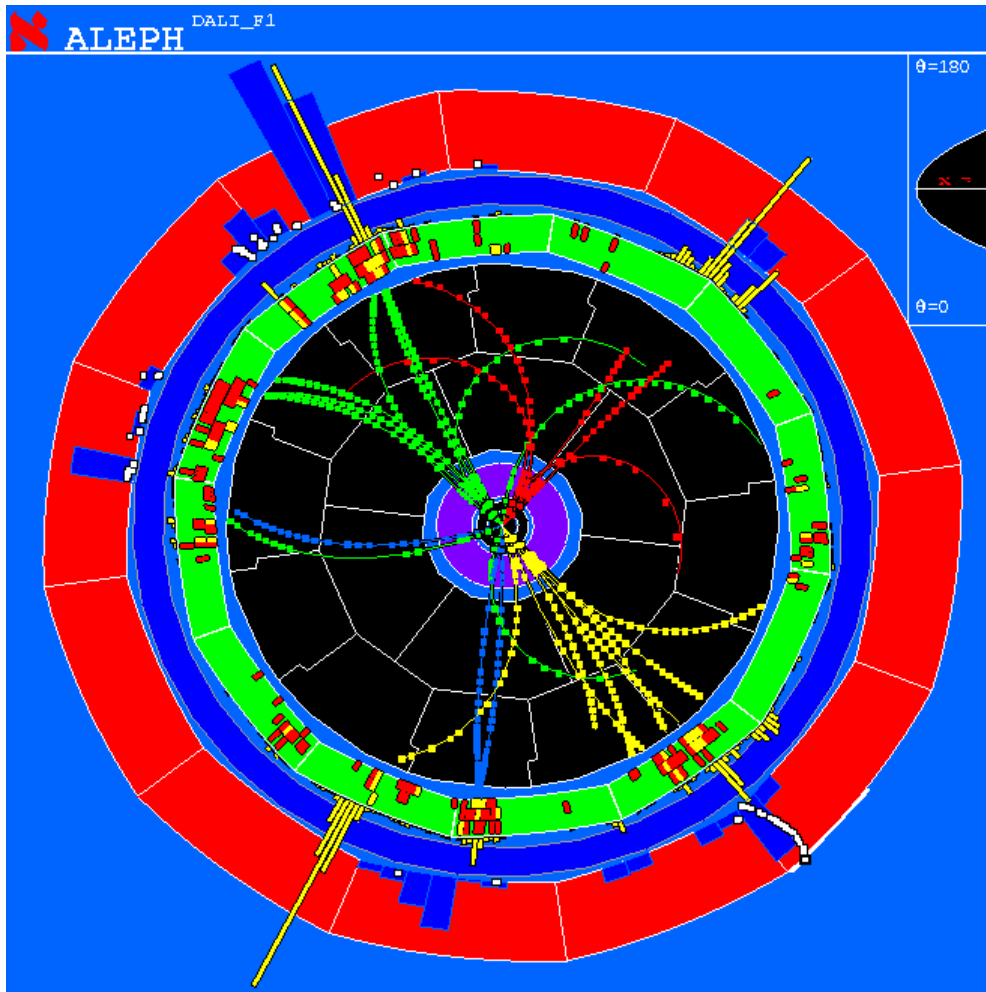
The energy loss as a function of particle momentum $P = Mc\beta\gamma$ IS depending on the particle's mass M .

By measuring the particle momentum (deflection in the magnetic field) and the energy loss of the particle, particles can be identified in certain momentum regions.



$$\frac{1}{\rho} \frac{dE}{dx} = -4\pi r_e^2 m_e c^2 Z_1^2 \frac{p^2 + M^2 c^2}{p^2} N_A \frac{Z}{A} \left[\ln \frac{2m_e c^2 F}{I} \frac{p^2}{M^2 c^2} - \frac{p^2}{p^2 + M^2 c^2} \right]$$

Energy Loss as a Function of the Momentum



Measure momentum by curvature of the particle track.

Find dE/dx by measuring the deposited charge along the track.

→ Particle ID

Range of Particles

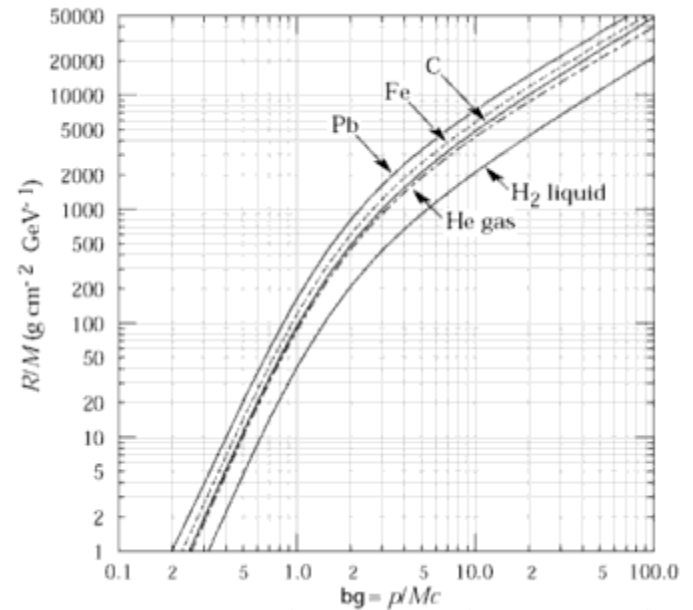
Particle of mass M and kinetic Energy E_0 enters matter and loses energy until it comes to rest at distance R .

$$R(E_0) = \int_{E_0}^0 \frac{-1}{dE/dx} dE$$

$$R(\beta_0 \gamma_0) = \frac{Mc^2}{\rho} \frac{1}{Z_1^2} \frac{A}{Z} f(\beta_0 \gamma_0)$$

$$\frac{\rho}{Mc^2} R(\beta_0 \gamma_0) = \frac{1}{Z_1^2} \frac{A}{Z} f(\beta_0 \gamma_0)$$

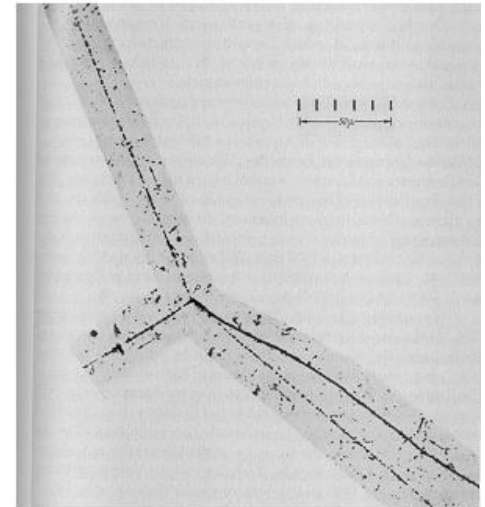
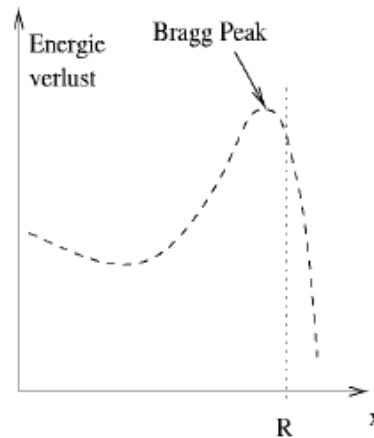
≈ Independent of the material



Bragg Peak: For $\beta\gamma > 3$ the energy loss is \approx constant (Fermi Plateau)

If the energy of the particle falls below $\beta\gamma = 3$ the energy loss rises as $1/\beta^2$

Towards the end of the track the energy loss is largest → Cancer Therapy.



Range of Particles

Average Range: Example

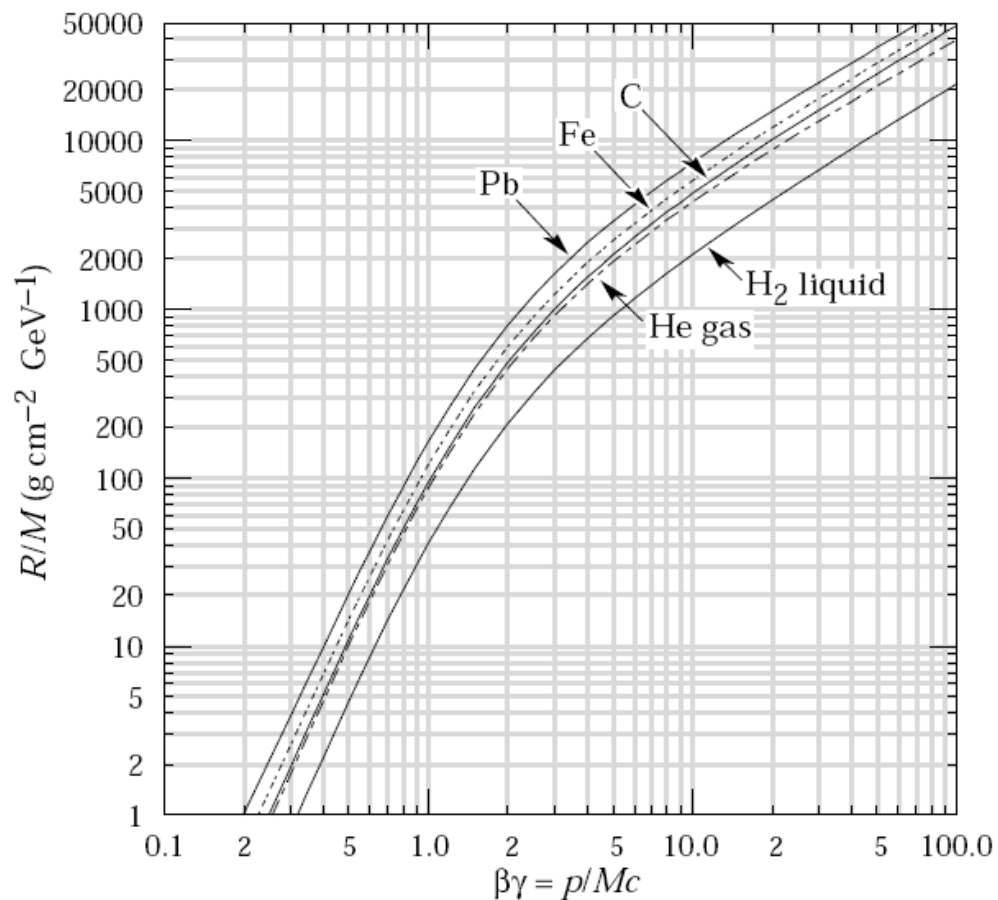
Kaon, $p=700$ MeV/c in Water:

$\rho=1$ [g/cm³], $Mc^2=494$ MeV \rightarrow

$\beta\gamma=1.42 \rightarrow \rho/Mc^2 R = 396$ g/cm² GeV

$\rightarrow R=195$ cm

In Pb: $\rho =11.35 \rightarrow R=17$ cm



Search for Hidden Chambers in the Pyramids

The structure of the Second Pyramid of Giza is determined by cosmic-ray absorption.

Luis W. Alvarez, Jared A. Anderson, F. El Bedwei, James Burkhard, Ahmed Fakhry, Adib Girgis, Amr Goneid, Fikhray Hassan, Dennis Iverson, Gerald Lynch, Zenab Miligy, Ali Hilmy Moussa, Mohammed-Sharkawi, Lauren Yazolino

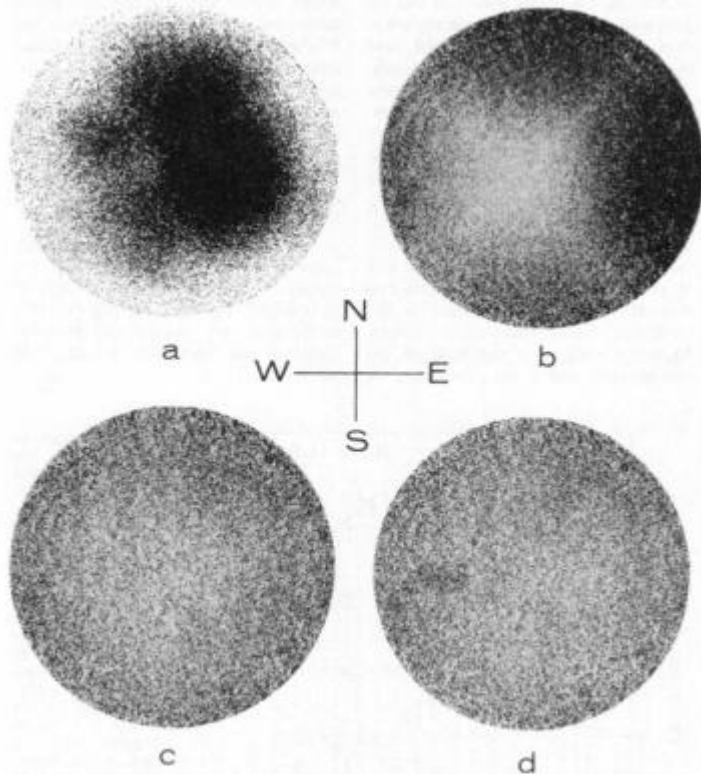


Fig. 13. Scatter plots showing the three stages in the combined analytic and visual analysis of the data and a plot with a simulated chamber, (a) Simulated "x-ray photograph" of uncorrected data, (b) Data corrected for the geometrical acceptance of the apparatus, (c) Data corrected for pyramid structure as well as geometrical acceptance, (d) Same as (c) but with simulated chamber, as in Fig. 12.

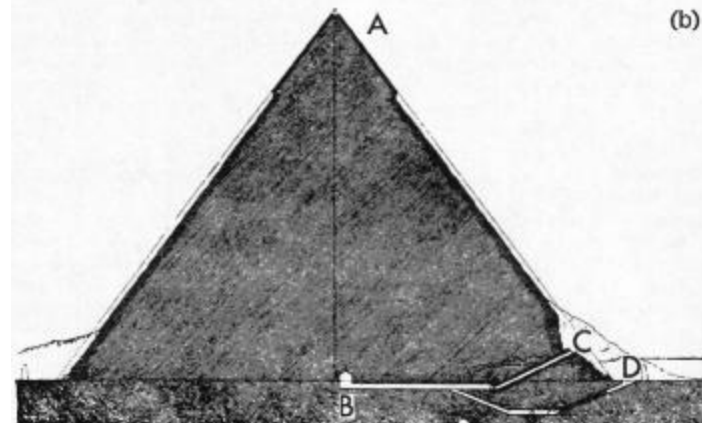
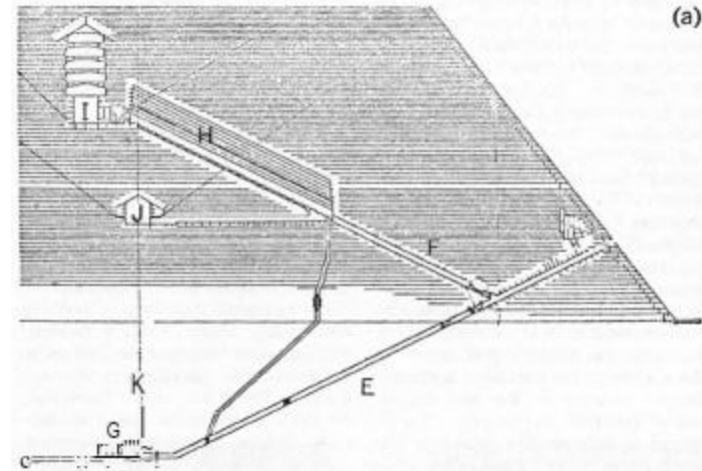
Fig. 2 (bottom right). Cross sections of (a) the Great Pyramid of Cheops and (b) the Pyramid of Chephren, showing the known chambers: (A) Smooth limestone cap, (B) the Belzoni Chamber, (C) Belzoni's entrance, (D) Howard-Vyse's entrance, (E) descending passageway, (F) ascending passageway, (G) underground chamber, (H-1) Grand Gallery, (I) King's Chamber, (J) Queen's Chamber, (K) center line of the pyramid.

6 FEBRUARY 1970



Luis Alvarez used the attenuation of muons to look for chambers in the Second Giza Pyramid → Muon Tomography

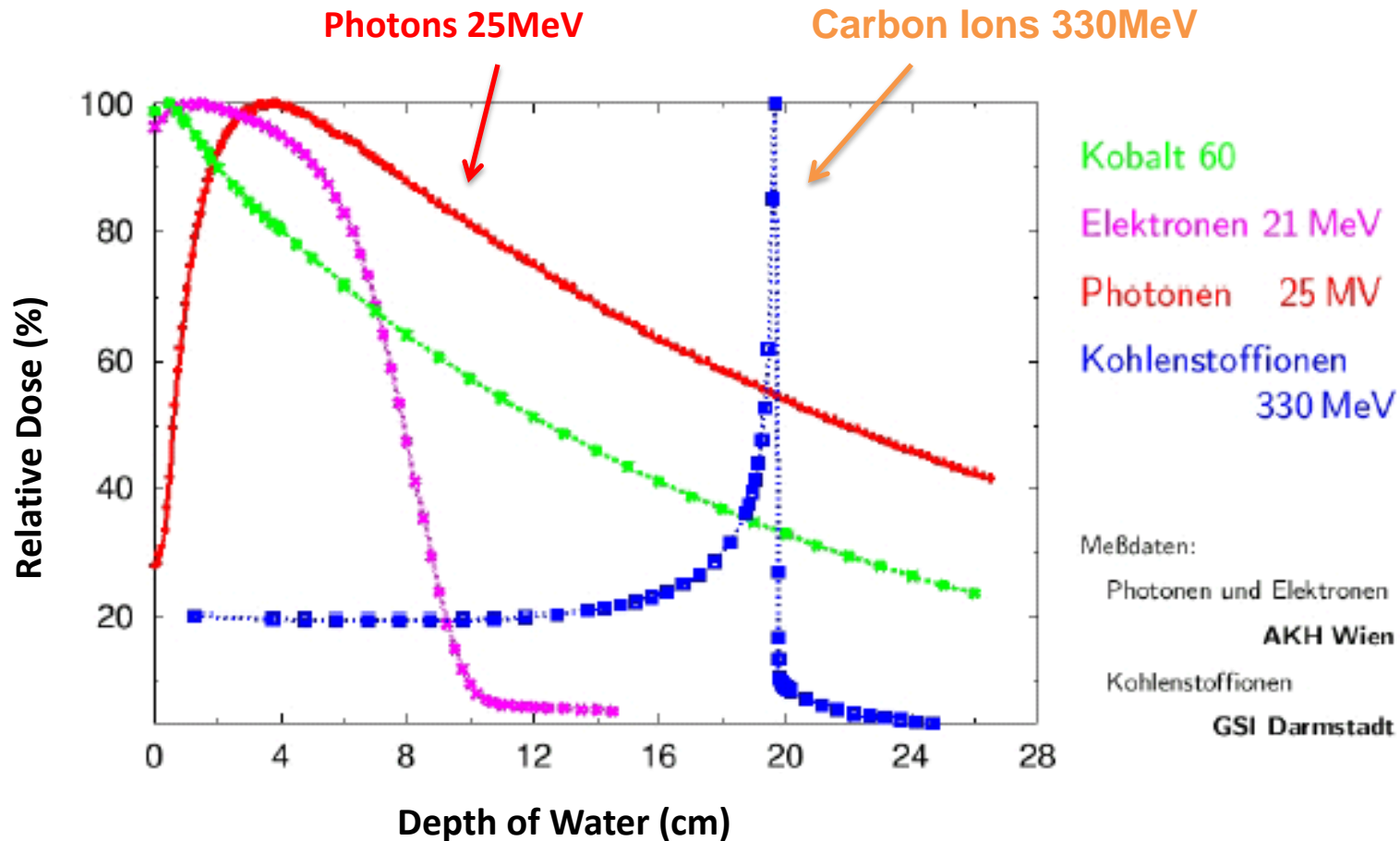
He proved that there are no chambers present.



Range of Particles

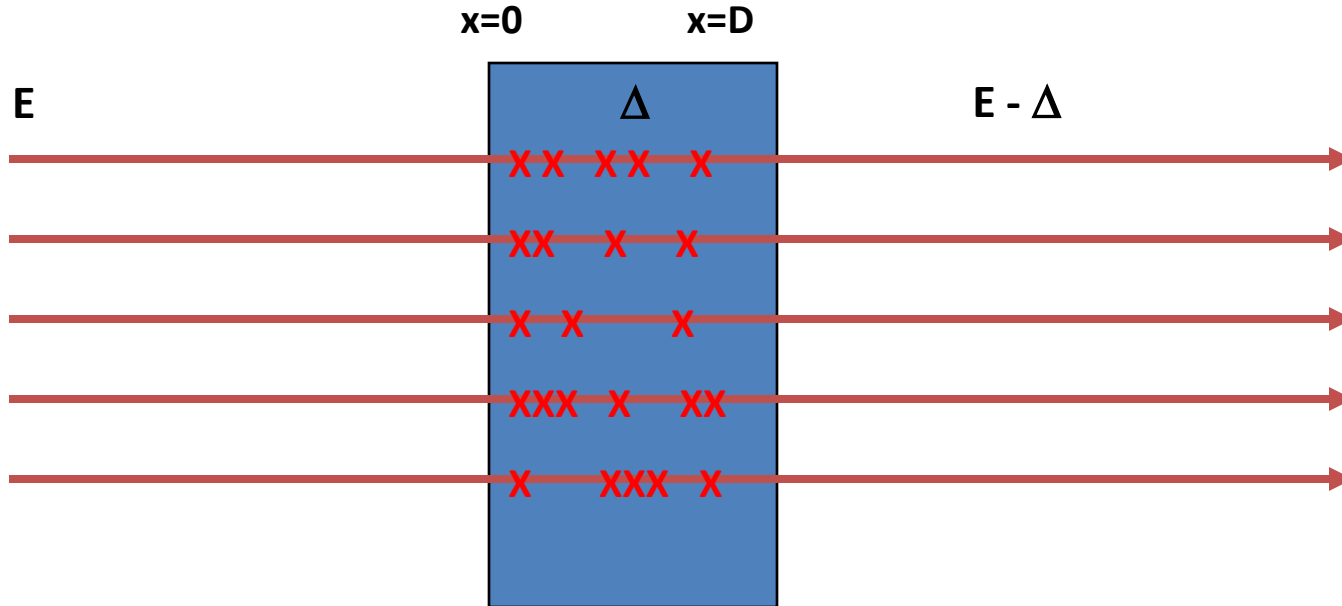
Average Range:

Towards the end of the track the energy loss is largest → Bragg Peak → Cancer Therapy



Fluctuation of Energy Loss

Up to now we have calculated the average energy loss. The energy loss is however a statistical process and will therefore fluctuate from event to event.



$P(\Delta) = ?$ Probability that a particle loses an energy Δ when traversing a material of thickness D

We have seen earlier that the probability of an interaction occurring between distance x and $x+dx$ is exponentially distributed

$$P(x)dx = \frac{1}{\lambda} \exp\left(-\frac{x}{\lambda}\right) dx \quad \lambda = \frac{A}{N_A \rho \sigma}$$

Probability for n Interactions in D

We first calculate the probability to find n interactions in D, knowing that the probability to find a distance x between two interactions is $P(x)dx = 1/\lambda \exp(-x/\lambda) dx$ with $\lambda = A/N_A\rho\sigma$

Probability to have no interaction between 0 und D:

$$P(x > D) = \int_D^\infty P(x_1)dx_1 = e^{-\frac{D}{\lambda}}$$

Probability to have one interaction at x_1 and no other interaction:

$$P(x_1, x_2 > D) = \int_D^\infty P(x_1)P(x_2 - x_1)dx_2 = \frac{1}{\lambda} e^{-\frac{D}{\lambda}}$$

Probability to have one interaction independently of x_1 :

$$\int_0^D P(x_1, x_2 > D) = \frac{D}{\lambda} e^{-\frac{D}{\lambda}}$$

Probability to have the first interaction at x_1 , the second at x_2 the n^{th} at x_n and no other interaction:

$$P(x_1, x_2 \dots x_n > D) = \int_D^\infty P(x_1)P(x_2 - x_1) \dots P(x_n - x_{n-1})dx_n = \frac{1}{\lambda^n} e^{-\frac{D}{\lambda}}$$

Probability for n interactions independently of $x_1, x_2 \dots x_n$

$$\int_0^D \int_0^{x_{n-1}} \int_0^{x_{n-1}} \dots \int_0^{x_1} P(x_1, x_2, \dots, x_n > D) dx_1 \dots dx_{n-1} = \frac{1}{n!} \left(\frac{D}{\lambda}\right)^n e^{-\frac{D}{\lambda}}$$

Probability for n Interactions in D

For an interaction with a mean free path of λ , the probability for n interactions on a distance D is given by

$$P(n) = \frac{1}{n!} \left(\frac{D}{\lambda} \right)^n e^{-\frac{D}{\lambda}} = \frac{\bar{n}^n}{n!} e^{-\bar{n}} \quad \bar{n} = \frac{D}{\lambda} \quad \lambda = \frac{A}{N_A \rho \sigma}$$

→ Poisson Distribution !

If the distance between interactions is exponentially distributed with an mean free path of λ → the number of interactions on a distance D is Poisson distributed with an average of $\bar{n}=D/\lambda$.

How do we find the energy loss distribution ?

If $f(E)$ is the probability to lose the energy E' in an interaction, the probability $p(E)$ to lose an energy E over the distance D ?

$$f(E) = \frac{1}{\sigma} \frac{d\sigma}{dE}$$

$$p(E) = P(1)f(E) + P(2) \int_0^E f(E-E')f(E')dE' + P(3) \int_0^E \int_0^{E'} f(E-E'-E'')f(E'')f(E')dE''dE' + \dots$$

$$F(s) = \mathcal{L}[f(E)] = \int_0^\infty f(E)e^{-sE}dE$$

$$\mathcal{L}[p(E)] = P(1)F(s) + P(2)F(s)^2 + P(3)F(s)^3 + \dots = \sum_{n=1}^{\infty} P(n)F(s)^n = \sum_{n=1}^{\infty} \frac{\bar{n}^n F^n}{n!} e^{-\bar{n}} = e^{\bar{n}(F(s)-1)} - 1 \approx e^{\bar{n}(F(s)-1)}$$

$$p(E) = \mathcal{L}^{-1} \left[e^{\bar{n}(F(s)-1)} \right] = \frac{1}{2\pi i} \int_{\sigma-i\infty}^{\sigma+i\infty} e^{\bar{n}(F(s)-1)+sE} ds$$

Probability for Energy Loss E in D

If \bar{n} is the average number of interactions in D and $F(s)$ is the Laplace transform for $f(E)$, giving the probability to lose the energy E in a collision, the probability $p(E)$ to lose the energy E in D is given by

$$p(E) = \mathcal{L}^{-1} \left[e^{\bar{n}(F(s)-1)} \right] = \frac{1}{2\pi i} \int_{\sigma-i\infty}^{\sigma+i\infty} e^{\bar{n}(F(s)-1)+sE} ds$$

Landau used the Rutherford scattering crosssection

$$\frac{d\sigma}{dE} = Z_1^2 r_e^2 \frac{2\pi m_e c^2}{\beta^2} \frac{1}{E^2} = \frac{k}{E^2} \quad \epsilon < E < E_{max}$$

The total crosssection, using a lower cutoff energy is then

$$\sigma = \int_{\epsilon}^{E_{max}} \frac{d\sigma}{dE} dE = k \left(\frac{1}{\epsilon} - \frac{1}{E_{max}} \right) \approx \frac{k}{\epsilon} \quad \bar{n} = \frac{N_A \rho Z_2 \sigma D}{A} = \frac{N_A \rho Z_2 k D}{A \epsilon}$$

The probability to lose and Energy E in a collision is then

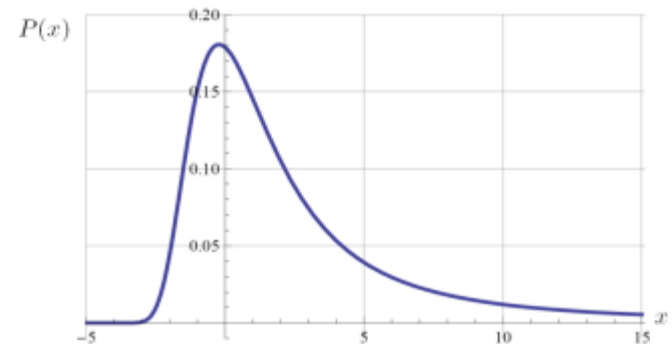
$$f(E) = \frac{1}{\sigma} \frac{d\sigma}{dE} = \frac{\epsilon}{E^2} \quad \epsilon < E < E_{max}$$

The Laplace transform of this probability density is

$$F(s) = \mathcal{L} \left[\frac{\epsilon}{E^2} \right] = \int_{\epsilon}^{\infty} \frac{\epsilon e^{-sE}}{E^2} dE \approx 1 + s\epsilon(1 - C_\gamma + \ln s\epsilon)$$

Which results in the energy loss distribution

$$p(E) = \frac{1}{2\pi i} \int_{\sigma-i\infty}^{\sigma+i\infty} \exp [\bar{n}s\epsilon (C_\gamma - 1 + \ln s\epsilon) + sE] ds$$



$C_\gamma = 0.577218... = \text{Euler constant}$

Landau Distribution

The previous expression can be rewritten as

$$P(x) = \frac{1}{2\pi i} \int_{\sigma-i\infty}^{\sigma+i\infty} e^{s \ln s + xs} ds = \frac{1}{\pi} \int_0^{\infty} e^{-t \ln t - xt} dt \quad x = \frac{E}{\bar{n}\epsilon} + C_\gamma - 1 - \ln \bar{n} \quad \bar{n} = \frac{N_A \rho Z_2 k D}{A \epsilon}$$

Landau determined the lower cutoff energy by requiring the average energy loss to be equal to the Bethe Bloch theory

$$\frac{dE}{dx} = -\frac{N_A \rho Z_2}{A} \int_{\epsilon}^{E_{max}} E \frac{d\sigma}{dE} dE = -\frac{N_A \rho Z_2 k}{A} \ln \frac{E_{max}}{\epsilon} = \text{Bethe-Bloch} = -2 \frac{N_A \rho Z_2 k}{A} \left(\ln \frac{E_{max}}{\epsilon} - \beta^2 \right)$$

which gives

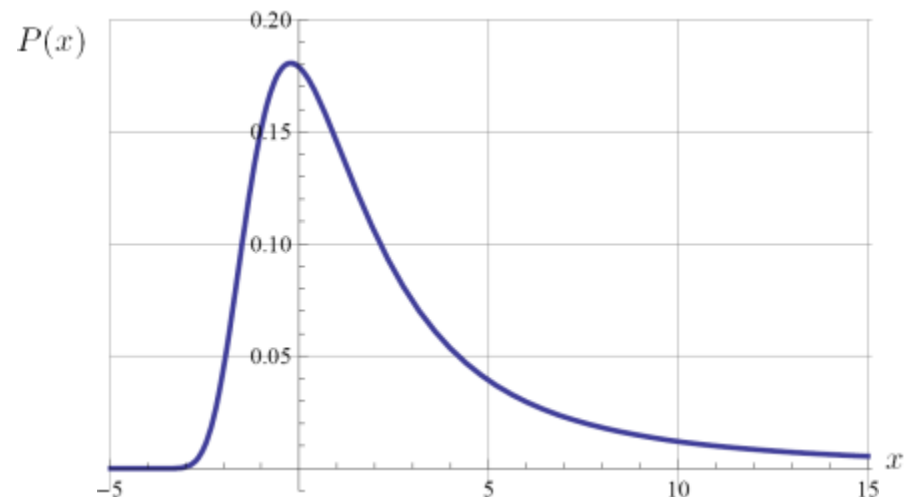
$$\ln \epsilon = \ln \frac{I^2}{E_{max}} + 2\beta^2$$

The Landau Distribution $P(x)$ has a peak at $x_0 = -0.2228 \sim C_\gamma - 0.8$ and a full width of half maximum of $\Delta x = 4.02$

$$E_{MP} = \bar{n}\epsilon(x_0 + 1 - C_\gamma + \ln \bar{n}) \approx \bar{n}\epsilon(0.2 + \ln \bar{n})$$

$$\Delta E_{FWHM} = 4.02 \bar{n}\epsilon = \frac{4.02 N_A \rho Z_2 k D}{A}$$

$$\frac{\Delta E_{FWHM}}{E_{MP}} = \frac{4.02}{0.2 + \ln \bar{n}}$$



Landau Distribution

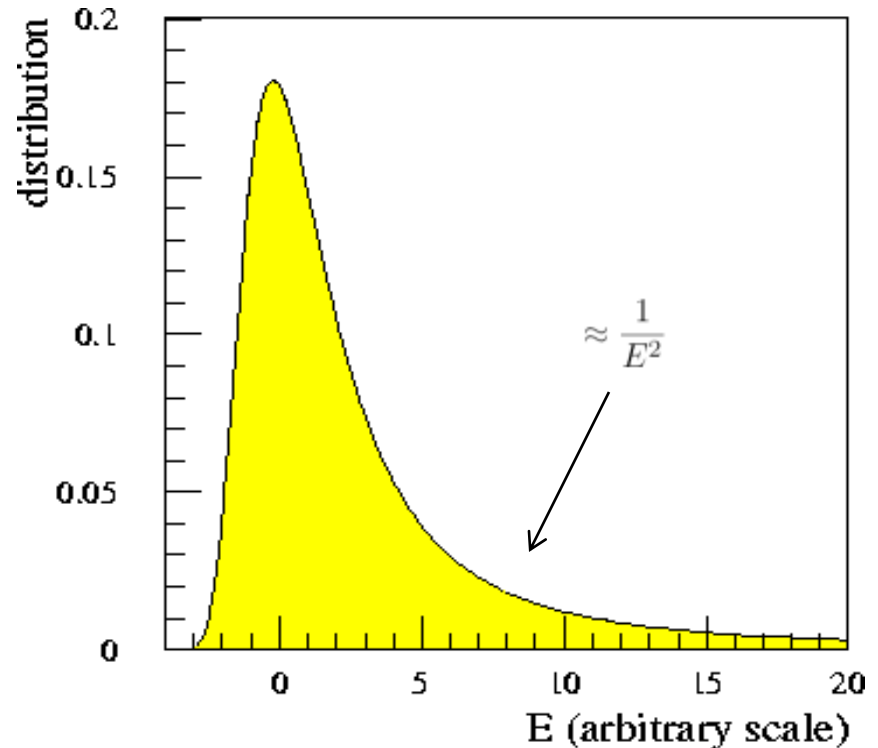
$p(E)$: Probability for energy loss E
in matter of thickness D .

Landau distribution is very
asymmetric.

Average and most probable
energy loss must be
distinguished !

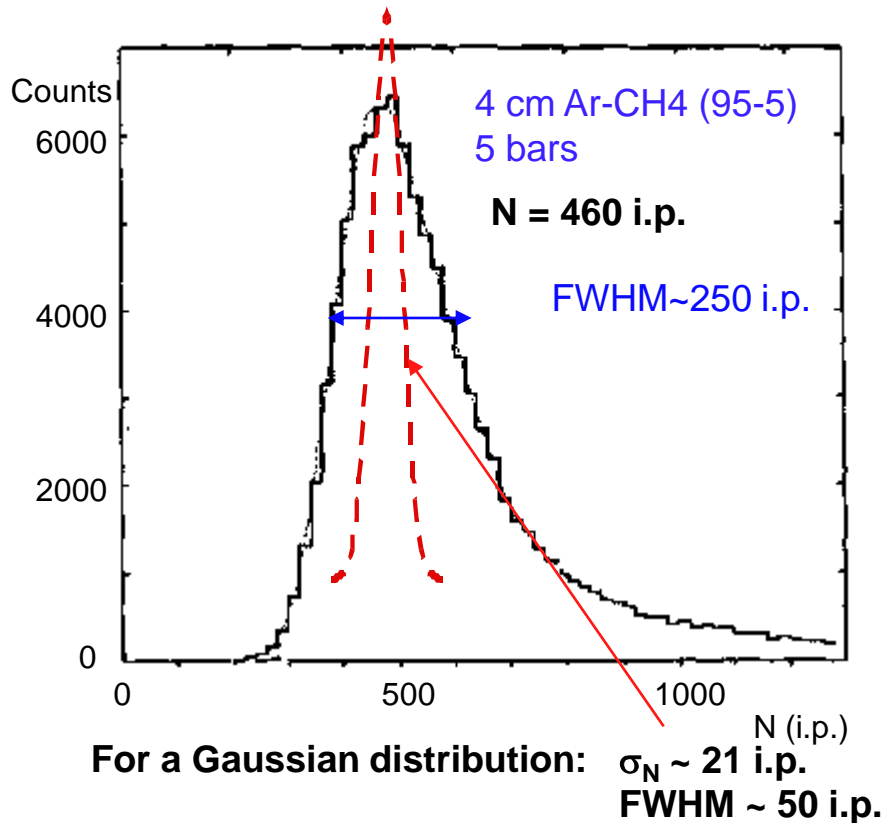
Measured Energy Loss is usually
smaller than the real energy loss:

3 GeV Pion: $E'_{\max} = 450\text{MeV} \rightarrow$ A
450 MeV Electron usually leaves
the detector.



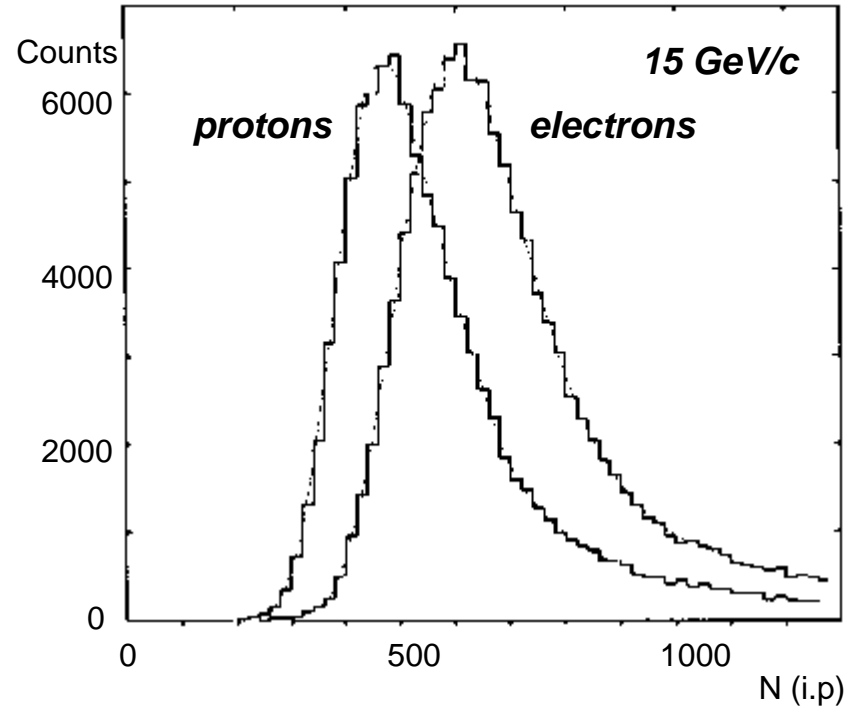
Landau Distribution

LANDAU DISTRIBUTION OF ENERGY LOSS:



PARTICLE IDENTIFICATION

Requires statistical analysis of hundreds of samples



I. Lehrs et al, Phys. Scripta 23(1981)727

Energy Loss Fluctuation

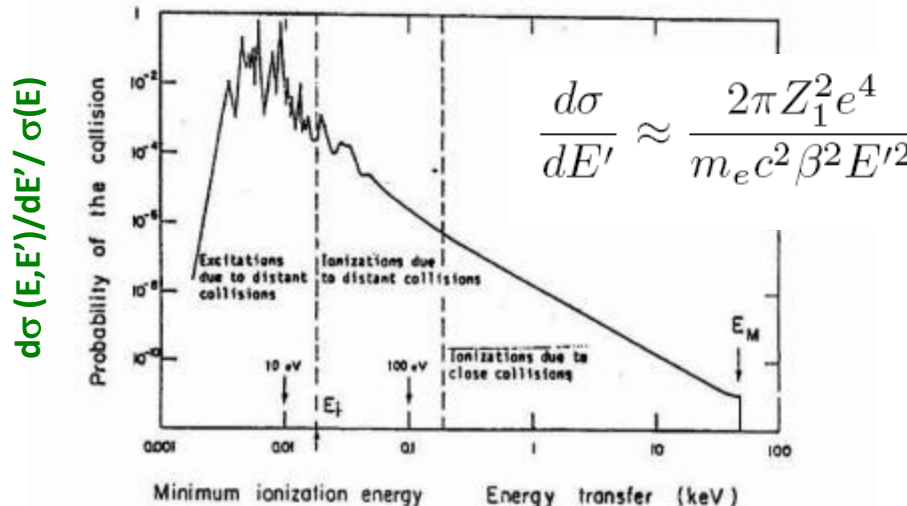
For calculation of the average energy loss, the integration

$$\frac{d\sigma_n(q)}{dq} = \frac{8\pi\alpha^2}{K^2} \left(\frac{M}{m}Z_1\right)^2 \frac{1}{q^3} |\epsilon_n(q)|^2 \quad \epsilon_n(q) = -\sum_{j=1}^{Z_2} \int e^{i(\vec{q}\vec{r}_j)} \psi_0(\vec{r}_j) \bar{\psi}_n(\vec{r}_j) d^3r_1 \dots d^3r_{Z_2}$$

$$\frac{dE}{dx}|_{E' < \eta} = -\sum_n (E_n - E_0) \int \frac{d\sigma_n(q)}{dq} dq = \sum_n E'_n \int \frac{d\sigma_n(q)}{dq} dq \quad E'_n < \eta$$

can be performed without the explicit calculation of ψ due to sum rules that must be satisfied.

For calculation of the differential energy loss and therefore the energy loss fluctuations, the explicit knowledge of ψ is necessary, which is however only explicitly possible for atomic Hydrogen



How do we find $P(E)$ for different Atoms and Molecules ?

Fermi Virtual Photon Method (FVP) or Photo Absorption Ionization Models (PAI, Allison & Cobb)

Fermi (Z. f. Physik 20. Oct 1924 !):

Über die Theorie des Stosses zwischen Atomen und elektrisch geladenen Teilchen
(On the theory of atomic collisions of charged particles).

Abstract: The electric field of a charged particle, which is flying by an atom, is harmonically decomposed and compared to light of the corresponding frequency distribution. We assume that the probability of atomic excitation and ionization by the passing particle is equal to atomic excitation and ionization of the equivalent radiation ...

Bethe 1930:

$$\epsilon_n(q) = - \sum_{j=1}^{Z_2} \int e^{i(\vec{q}\vec{r}_j)} \psi_0(\vec{r}_j) \bar{\psi}_n(\vec{r}_j) d^3r_1 \dots d^3r_{Z_2}$$
$$e^{i\vec{q}\vec{r}_j} \approx 1 + i\vec{q}\vec{r}_j + \dots \quad \epsilon_n(q) = i\vec{q} \int \sum_{j=1}^{Z_2} \vec{r}_j \psi_0(r_j) \bar{\psi}_n(r_j) d^3r_1 \dots d^3r_{Z_2} = i\vec{q} \vec{x}_{0n}$$

“...Thus, for small collision vectors q (i.e. small changes in momentum of the colliding particle), the collision probability is proportional to the square of the coordinate matrix, i.e., proportional to the optical transition probability for the respective excitation of the atom ...” (Bethe, 1930)

Knowing the optical photoabsorption cross section, if which detailed measurements are available !

Fermi Virtual Photon Method (FVP) or Photo Absorption Ionization Models (PAI, Allison & Cobb)

Allison & Cobb, Ann. Rev. Nucl. Part. Sci. 1980, 253-298:

First they calculate the energy loss of a particle passing a homogenous medium with dielectric permittivity $\epsilon = \epsilon_1 + i\epsilon_2$. The result will therefore contain the density effect, i.e. the reduction of the energy loss due to polarization of the medium. Then they make a model for ϵ_2 by using the atomic photo absorption crosssection.

Particle of charge e_0 and velocity v (E.g. Landau and Lifshitz):

$$\begin{aligned} \operatorname{div} H &= 0 & \operatorname{rot} E &= -\frac{1}{c} \frac{\partial H}{\partial t} & \operatorname{div} \epsilon E &= 4\pi\rho & \operatorname{rot} H &= \frac{1}{c} \frac{\partial \epsilon E}{\partial t} + \frac{4\pi}{c} j \\ \rho &= e_0 \delta^3(r - \beta ct) & j &= \beta c\rho & \epsilon &= \epsilon_1 + i\epsilon_2 \end{aligned}$$

$$\frac{dE}{dx} = -\frac{2e_0^2}{\beta^2\pi} \int_0^\infty d\omega \int_{\omega/v}^\infty dk \left[\omega k \left(\beta^2 - \frac{\omega^2}{k^2 c^2} \right) \operatorname{Im} \left(\frac{1}{-k^2 c^2 + \epsilon \omega^2} \right) - \frac{\omega}{k c^2} \operatorname{Im} \left(\frac{1}{\epsilon} \right) \right]$$

A plane light wave travelling along x is attenuated in in the medium if the imaginary part of ϵ is different from zero.

$$E(x) \sim e^{ikx} \quad I(x) \sim E(x)^2 = e^{2ikx} \quad k = \frac{\omega}{v} = \frac{\omega}{c} \sqrt{\epsilon} \approx \frac{\omega}{c} \left(1 + i \frac{\epsilon_2}{2} \right) \quad \epsilon_1 - 1, \epsilon_2 \ll q \quad I(x) \sim e^{-\frac{\omega}{c} \epsilon_2(\omega) x}$$

A photon interacting with material with an atomic crosssection σ has a mean free path of $\lambda = A/N_A \rho \sigma$. The probability of interacting after travelling a distance x is

$$P(x) = \frac{1}{\lambda} e^{-x/\lambda} \sim e^{-N_A \rho \sigma(\omega) x/A} \quad \rightarrow \quad \frac{N_A \rho}{A} \sigma(\omega) = \frac{\omega}{c} \epsilon_2(\omega)$$

Fermi Virtual Photon Method (FVP) or Photo Absorption Ionization Models (PAI, Allison & Cobb)

This defines the imaginary part of the dielectric constant in terms of the atomic photoabsorption cross section for small k. For large k and expression is introduced in order to satisfy the Bethe Sum rule:

Small k: $\epsilon_2(\omega) = \frac{N_{A\rho}}{A} \frac{c}{\omega} \sigma(\omega)$ **Large k:** $\epsilon_2(k, \omega) = C \delta(\omega - k^2/2m_e)$ $\int \omega \epsilon_2(k, \omega) d\omega = \frac{2\pi^2 e_0^2 N_{A\rho} Z_2}{A m_e}$

The real part is uniquely related to the imaginary part by the Kramers-Kronig relations

$$\epsilon_1(\omega) - 1 = \frac{2}{\pi} \text{P} \int_0^\infty \frac{x \epsilon_2(x)}{x^2 - \omega^2} dx$$

We are therefore in a position to integrate the expression over k and have

$$\frac{dE}{dx} = - \int_0^\infty d\omega \frac{e_0^2}{\beta^2 c^2 \pi^2} \left[\frac{N_{A\rho}}{A} \sigma(\omega) \ln[(1 - \beta^2 \epsilon_1)^2 + \beta^4 \epsilon_2^2]^{-1/2} + \omega \left(\beta^2 - \frac{\epsilon_1}{|\epsilon|^2} \right) \Theta + \frac{N_{A\rho}}{A} \sigma(\omega) \ln \left(\frac{2m_e \beta^2 c^2}{\hbar \omega} \right) + \frac{1}{\omega} \int_0^\omega \frac{\sigma(\omega')}{Z_2} d\omega' \right]$$

$$\Theta = \arg(1 - \epsilon_1 \beta^2 + i \epsilon_2 \beta^2) = \frac{\pi}{2} - \arctan \frac{1 - \epsilon_1 \beta^2}{\epsilon_2 \beta^2}$$

We can then reinterpret this equation in terms of a number of discrete collisions with energy transfer $E = \hbar \omega$ and find the differential cross section for losing an energy E in a single collision

$$\frac{d\sigma}{dE} = \frac{\alpha}{\beta^2 \pi} \frac{\sigma(E)}{EZ} \ln[(1 - \beta^2 \epsilon_1)^2 + \beta^4 \epsilon_2^2]^{-1/2} + \frac{\alpha}{\beta^2 \pi} \frac{A}{N_{A\rho} Z_2 \hbar c} \left(\beta^2 - \frac{\epsilon_1}{|\epsilon|^2} \right) \Theta + \frac{\alpha}{\beta^2 \pi} \frac{\sigma(E)}{EZ_2} \ln \left(\frac{2m_e \beta^2 c^2}{E} \right) + \frac{\alpha}{\beta^2 \pi} \frac{1}{E^2} \int_0^E \frac{\sigma(E')}{Z_2} dE'$$

This is the model included in GEANT and HEED.

Fermi Virtual Photon Method (FVP) or Photo Absorption Ionization Models (PAI, Allison & Cobb)

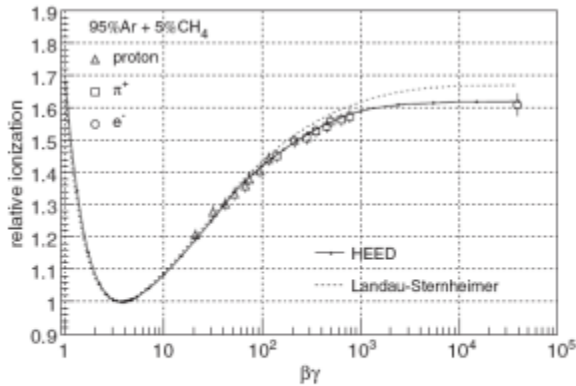


Fig. 7. The simulations of experiment [62] by HEED (truncated means by PAIR) and by the Landau-Sternheimer theory (the most probable ionization) compared with the normalized experimental data.

482

I.B. Smirnov / Nuclear Instruments and Methods in Physics Research A 554 (2005) 474–493

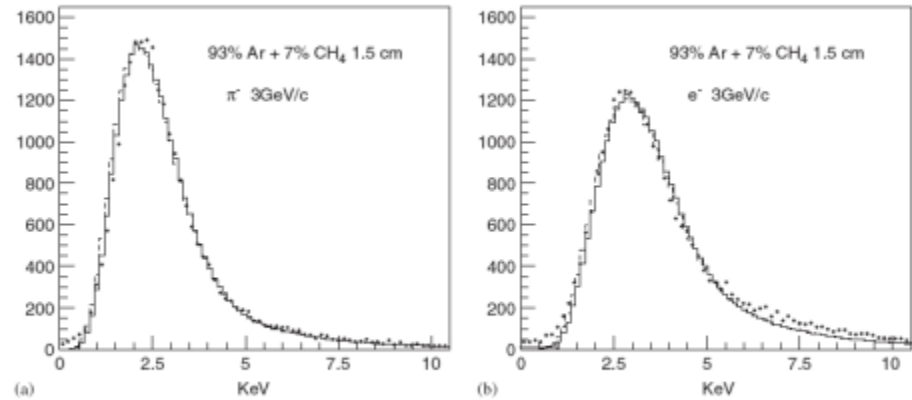
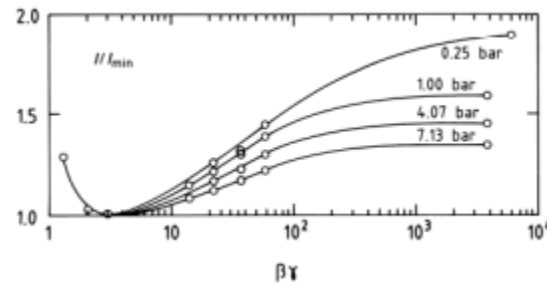


Fig. 3. The experimental (points) and theoretical “ionization loss” distributions expressed in energy units. Histograms drawn by solid lines are obtained by the PAIR model, dashed lines (they are practically coincide with solid ones) are by the PAI model.



Density Effect

The PAI model together with the good knowledge of the Photoabsorption Crosssections allows the accurate calculation of primary ionization & the associated fluctuations (Straggling functions)

Cherenkov Radiation

If we set the imaginary part of the dielectric permittivity to zero, the energy loss in the PAI expression doesn't become zero – there is a term left, which describes the Cherenkov Radiation → This is a classic effect described by Maxwell's equations

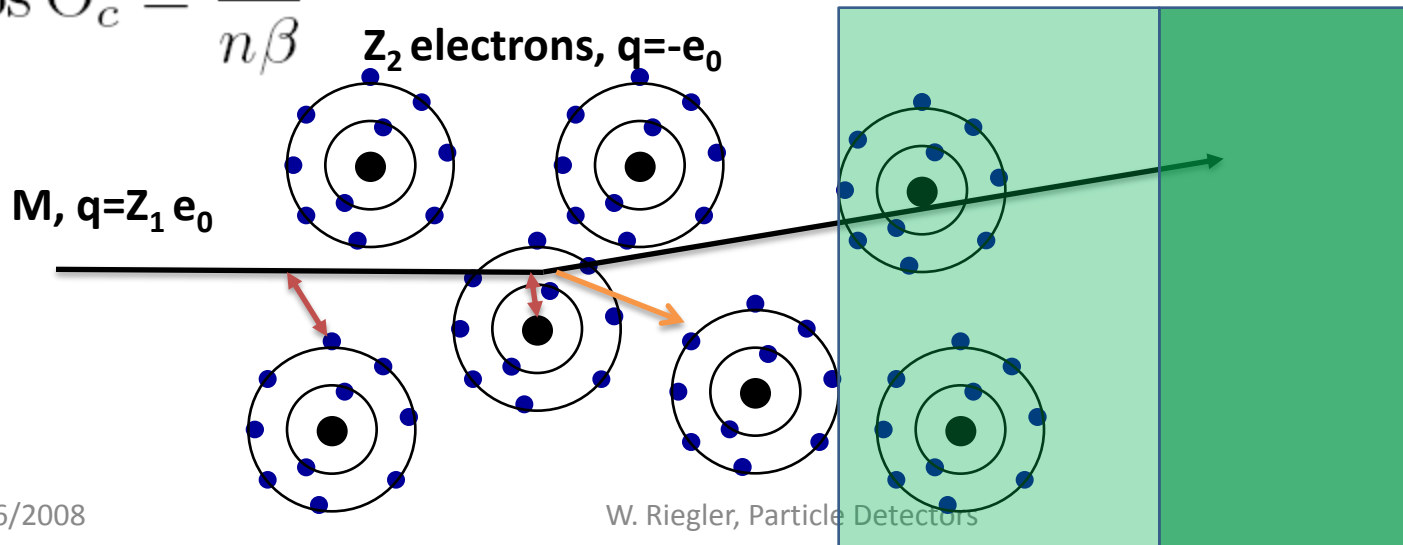
$$\frac{d\sigma}{dE} = \frac{\alpha}{\beta^2 \pi} \frac{A}{N_{Ap} Z_2 \hbar c} \left(\beta^2 - \frac{1}{\epsilon_1} \right) \rightarrow \frac{N_{Ap} Z_2}{A} \frac{d\sigma}{d\omega} \frac{d\omega}{dE} = \frac{\alpha}{c} \left(1 - \frac{1}{\beta^2 n^2} \right) \quad n = \sqrt{\epsilon_1} \quad E = \hbar \omega$$

$$\frac{dE}{dx d\omega} \frac{1}{\hbar} = \frac{\alpha}{c} \left(1 - \frac{1}{\beta^2 n^2} \right) \rightarrow \frac{dN}{dx d\lambda} = \frac{2\pi\alpha}{\lambda^2} \left(1 - \frac{1}{\beta^2 n^2} \right) \quad \omega = \frac{2\pi c}{\lambda}$$

N is the number of Cherenkov Photons emitted per cm of material. The expression is in addition proportional to Z_1^2 of the incoming particle.

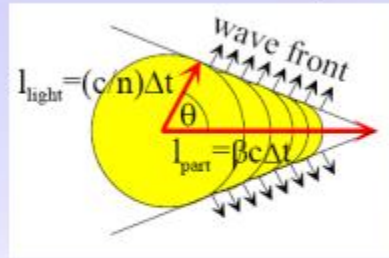
This radiation is emitted if the velocity of the particle is larger than the velocity of light in the medium $v=c/n$ (shock wave). It is emitted at the characteristic angle θ_c .

$$\cos \Theta_c = \frac{1}{n\beta}$$



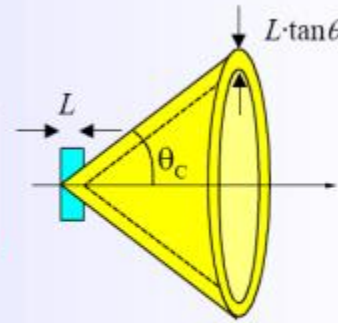
Cherenkov Radiation

with velocity $\beta \geq \beta_{thr} = \frac{1}{n}$ n : refractive index



$$\cos \theta_C = \frac{1}{n\beta}$$

with $n = n(\lambda) \geq 1$



■ $\beta_{thr} = \frac{1}{n} \rightarrow \theta_C \approx 0$ Cherenkov threshold

■ $\theta_{max} = \arccos \frac{1}{n}$ 'saturated' angle ($\beta=1$)

If the velocity of a charged particle is longer than the velocity of light in the medium $v > \frac{c}{n}$ (n ... refractive index of material) it emits 'Cherenkov' radiation at a characteristic angle of $\cos \theta_C = \frac{1}{n\beta}$ ($\beta = \frac{v}{c}$)

$$\frac{dN}{dx} \sim 2\pi d z_1^2 \left(1 - \frac{1}{\beta^2 n^2}\right) \frac{\lambda_2 - \lambda_1}{\lambda_2 \cdot \lambda_1}$$

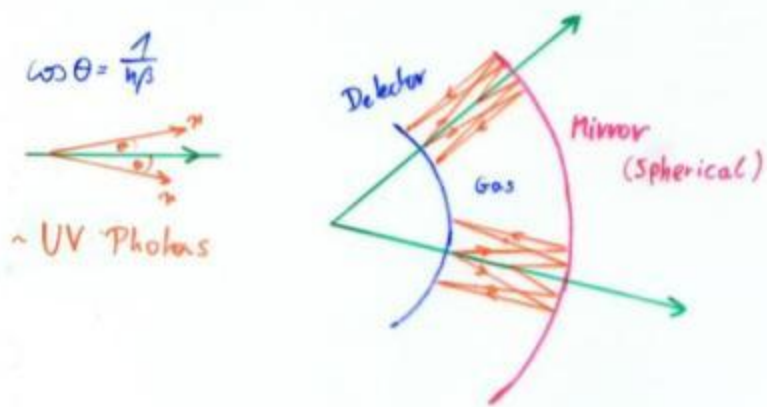
= Number of emitted photons/length with λ between λ_1 and λ_2

With $\lambda_1 = 400\text{nm}$ $\lambda_2 = 700\text{nm}$

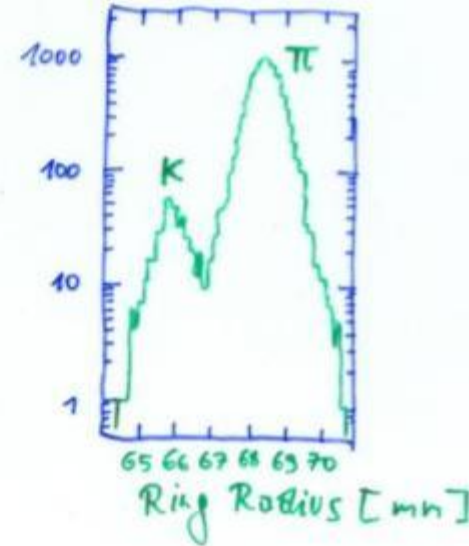
$$\frac{dN}{dx} = 490 \left(1 - \frac{1}{\beta^2 n^2}\right) \left[\frac{1}{\text{cm}}\right]$$

Material	$n-1$	β threshold	γ threshold
solid Sodium	3.22	0.24	1.029
lead glass	0.67	0.60	1.25
water	0.33	0.75	1.52
Silica aerogel	0.025-0.075	0.93-0.976	2.7 - 4.6
air	$2.93 \cdot 10^{-4}$	0.9997	41.2
He	$3.3 \cdot 10^{-5}$	0.99997	123

Ring Imaging Cherenkov Detector



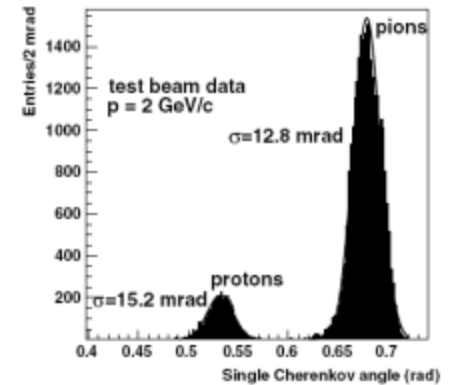
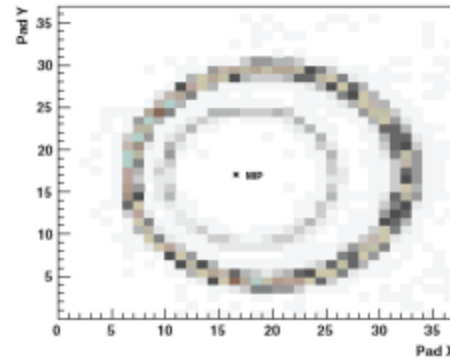
200 GeV/c K, π



Resolution $\frac{\Delta \eta}{\eta} = \gamma^2 \beta^3 n \Delta \theta \frac{1}{\sqrt{N_{ph} L}} \quad \left(\gamma = \frac{1}{\sqrt{1-\beta^2}} \right)$

Angle Measurement Accuracy

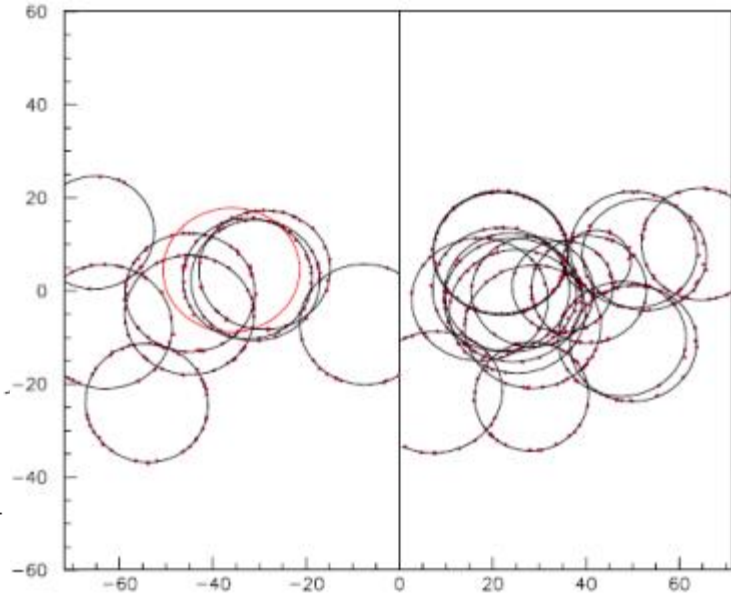
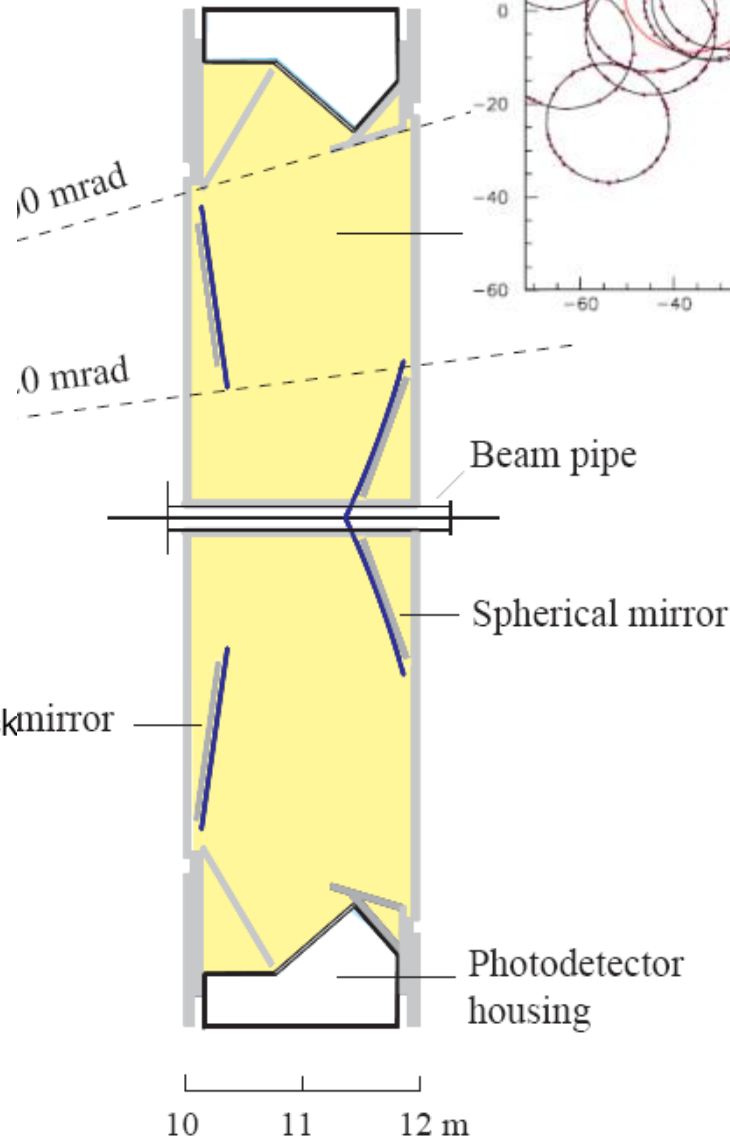
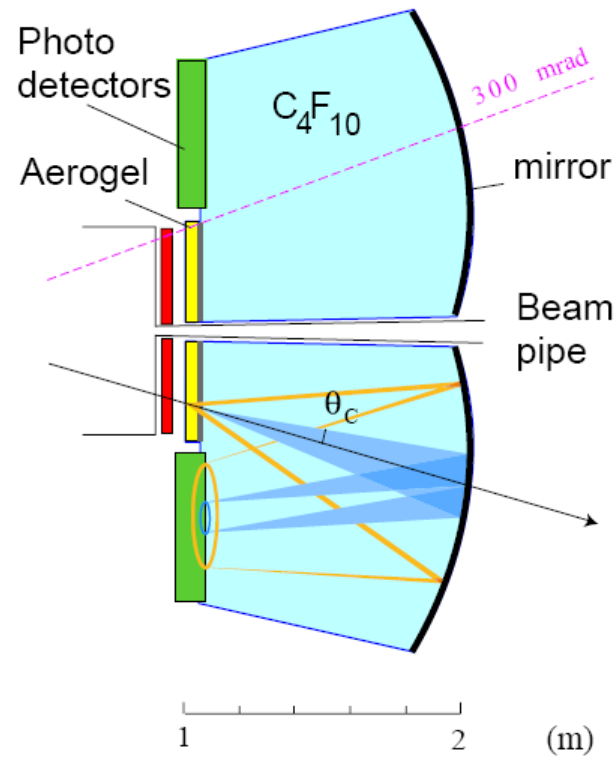
Photon Statistics



medium	n	θ_{\max} (deg.)	N_{ph} ($eV^{-1} cm^{-1}$)
air*	1.000283	1.36	0.208
isobutane*	1.00127	2.89	0.941
water	1.33	41.2	160.8
quartz	1.46	46.7	196.4

There are only 'a few' photons per event \rightarrow one needs highly sensitive photon detectors to measure the rings !

LHCb RICH



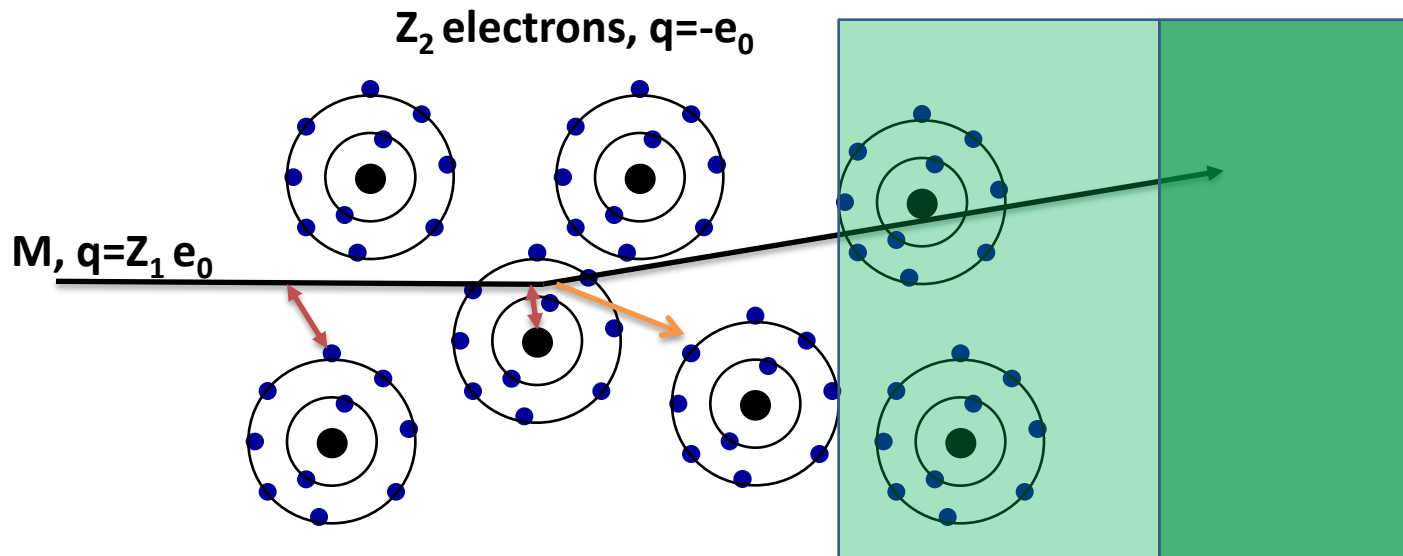
Bremsstrahlung

A charged particle of mass M and charge $q=Z_1e$ is deflected by a nucleus of charge Ze which is partially 'shielded' by the electrons. During this deflection the charge is 'accelerated' and it therefore radiated \rightarrow Bremsstrahlung.

From Bethe's theory we have seen that the elastic scattering off the Nucleus is given by

$$\epsilon_0(q) = Z_2 - \sum_{j=1}^{Z_2} \int e^{i(\vec{q}\vec{r}_j)} \psi_0^2(\vec{r}_j) d^3r_1 \dots d^3r_{Z_2} = Z_2 - F \quad \frac{d\sigma}{d\Omega} = \left(\frac{1}{4\pi\epsilon_0} \frac{Z_1(Z_2 - F)e_0^2}{2pv} \right)^2 \frac{1}{\sin^4 \theta/2}$$

Where $F(q)$ describes the partial shielding of the nucleus by the electrons. Effective values for F are used in the following expressions.



Bremsstrahlung, Classical



$$\frac{d\sigma'}{d\Omega} = \left(\frac{2Z_1 Z_2 e^2}{4\pi\epsilon_0 p \cdot v} \right)^2 \frac{1}{(2\sin(\frac{\theta}{2}))^4} \quad p = M v \gamma$$

"Rutherford Scattering"

Written in Terms of Momentum Transfer $Q^2 = 2p^2(1 - \cos\theta)$

$$\frac{d\sigma'}{dQ} = 8\pi \left(\frac{Z_1 Z_2 e^2}{4\pi\epsilon_0 \beta c} \right)^2 \cdot \frac{1}{Q^3}$$



$$Q = |\vec{p} - \vec{p}'|$$

$\lim_{\omega \rightarrow 0} \frac{dI}{d\omega} \sim \frac{2}{3\pi} \frac{Z_1^2 e^2}{M^2 c^3} \frac{1}{4\pi\epsilon_0} Q^2$ *From Maxwell's eq (Jackson)*
Radiated Energy between $\omega, \omega + d\omega$

$$\frac{dE}{dx} = \frac{N_A g}{A} \cdot \int_0^{Q_{max}} \int_{Q_{min}} dQ \frac{dI}{d\omega} \cdot \frac{d\sigma'}{dQ} \quad , \quad Q_{max} = \frac{E}{\hbar}$$

$$\frac{dE}{dx} = \frac{N_A g}{A} \cdot \frac{16}{3} d \cdot Z^2 \cdot \left(\frac{Z_1^2 e^2}{4\pi\epsilon_0 M c^2} \right)^2 \cdot E \cdot \ln \frac{Q_{max}}{Q_{min}}$$

$$d = \frac{e^2}{4\pi\epsilon_0 \hbar c} \sim \frac{1}{137}$$

A charged particle of mass M and charge $q=Z_1 e$ is deflected by a nucleus of Charge Ze.

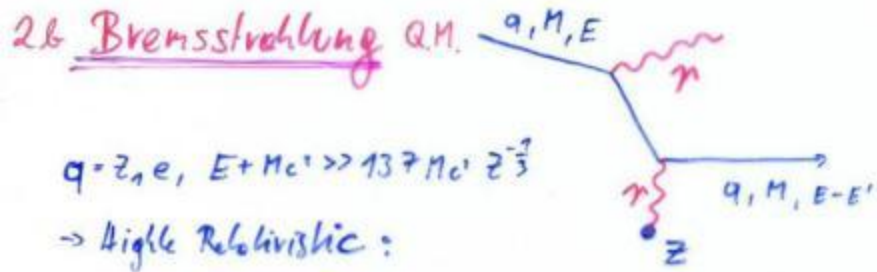
Because of the acceleration the particle radiated EM waves \rightarrow energy loss.

Coulomb-Scattering (Rutherford Scattering) describes the deflection of the particle.

Maxwell's Equations describe the radiated energy for a given momentum transfer.

$\rightarrow dE/dx$

Bremsstrahlung, QM



$$\frac{d\sigma(E, E')}{dE'} = 4Z^2 Z_1^4 \left(\frac{1}{4\pi\epsilon_0} \frac{e^2}{Mc^2} \right)^2 \left(\frac{1}{E'} \right) F(E, E')$$

$$F(E, E') = \left[1 + \left(1 - \frac{E'}{E + Mc^2} \right)^2 - \frac{2}{3} \left(1 - \frac{E'}{E + Mc^2} \right) \right] \ln 183 Z^{-2/3} + \frac{1}{9} \left(1 - \frac{E'}{E + Mc^2} \right)$$

$$\frac{dE}{dx} = - \frac{N_A \rho}{A} \int_0^E E' \frac{d\sigma}{dE'} dE' \approx 4Z^2 Z_1^4 \left(\frac{1}{4\pi\epsilon_0} \frac{e^2}{Mc^2} \right)^2 E \left[\ln 183 Z^{-2/3} + \frac{1}{18} \right]$$

$$\underline{\underline{\frac{dE}{dx} = - \frac{N_A \rho}{A} 4Z^2 Z_1^4 \left(\frac{1}{4\pi\epsilon_0} \frac{e^2}{Mc^2} \right)^2 E \ln(183 Z^{-2/3})}}$$

$$E(x) = E_0 e^{-\frac{x}{X_0}} \quad X_0 = \frac{A}{4Z^2 N_A \rho \left(\frac{1}{4\pi\epsilon_0} \frac{e^2}{Mc^2} \right)^2 \ln 183 Z^{-2/3}}$$

X_0 ... Radiation length

Proportional to Z^2/A of the Material.

Proportional to Z_1^4 of the incoming particle.

Proportional to ρ of the material.

Proportional $1/M^2$ of the incoming particle.

Proportional to the Energy of the Incoming particle →

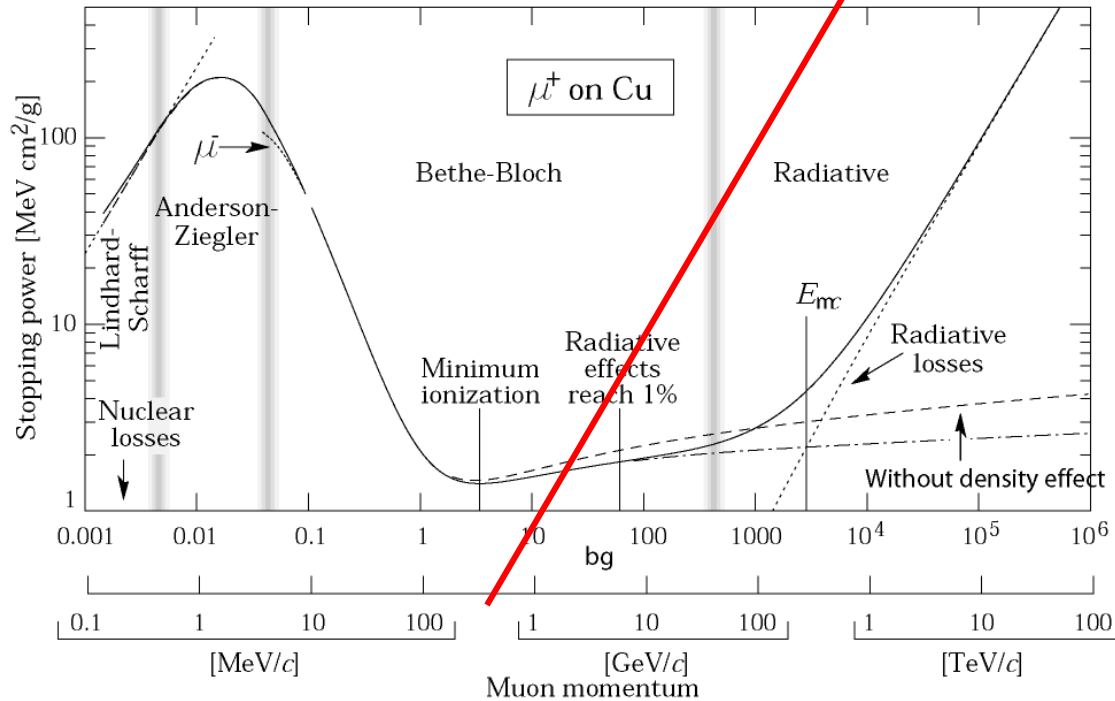
$E(x) = \text{Exp}(-x/X_0)$ – ‘Radiation Length’

$$X_0 \propto M^2 A / (\rho Z_1^4 Z^2)$$

X_0 : Distance where the Energy E_0 of the incoming particle decreases $E_0 \text{Exp}(-1) = 0.37 E_0$.

Critical Energy

such as copper to about 1% accuracy for energies between about 6 MeV and 6 GeV



Electron Momentum 5 50 500 MeV/c

Critical Energy: If dE/dx (Ionization) = dE/dx (Bremsstrahlung)

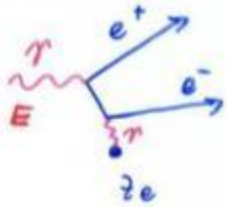
Myon in Copper: $p \approx 400\text{GeV}$

Electron in Copper: $p \approx 20\text{MeV}$

For the muon, the second lightest particle after the electron, the critical energy is at 400GeV.

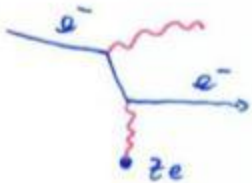
The EM Bremsstrahlung is therefore only relevant for electrons at energies of past and present detectors.

Pair Production, QM



$$\gamma + \text{Nucl.} \rightarrow e^+ + e^- + \text{Nucl.}$$

The Diagram is very similar to Bremsstrahlung



$e^- + \text{Nucl.} \rightarrow \gamma + e^- + \text{Nucl.}$
 Crossing Symmetry: bring particle to the other side and make it the anti-particle \rightarrow 'same' correction ...

$$\frac{d\sigma(E, E')}{dE'} = 4\alpha Z^2 v_0^2 \frac{1}{E} G(E, E') \quad E \gg 137 m_e c^2 Z^{-1/3}$$

$$G(E, E') = \left[\left(\frac{E'+m_e c^2}{E} \right)^2 \left(1 - \frac{E'+m_e c^2}{E} \right)^2 + \frac{2}{3} \frac{E'+m_e c^2}{E} \left(1 - \frac{E'+m_e c^2}{E} \right) \ln \frac{E}{E'} - \frac{1}{3} \frac{E'+m_e c^2}{E} \left(1 - \frac{E'+m_e c^2}{E} \right) \right]$$

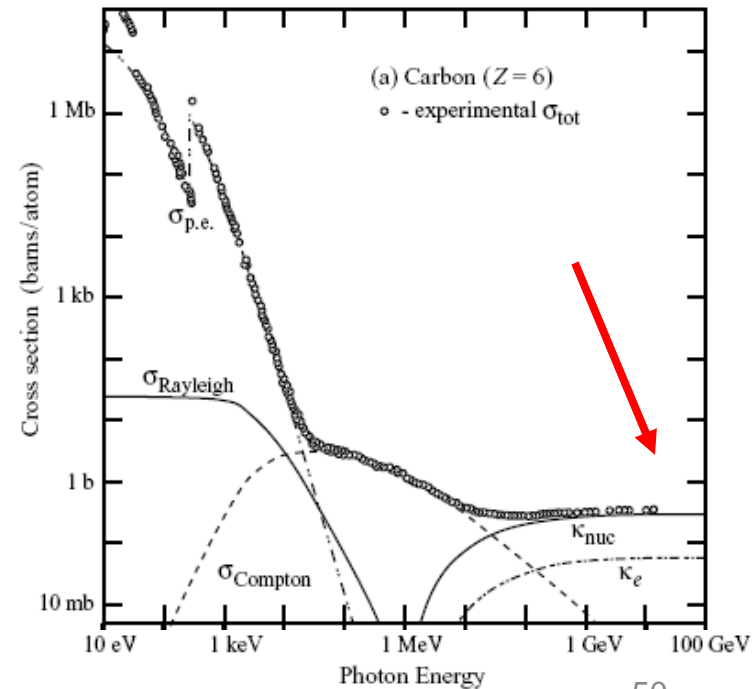
$$\sigma = \int_0^{E-2m_e c^2} \frac{d\sigma}{dE'} dE' = 4\alpha Z^2 v_0^2 \cdot \frac{7}{3} \ln 183 Z^{-1/3}$$

$$P(x) = \frac{1}{\lambda} e^{-x/\lambda} \quad \lambda = \frac{A}{9 N_A \sigma} = \frac{9}{7} X_0$$

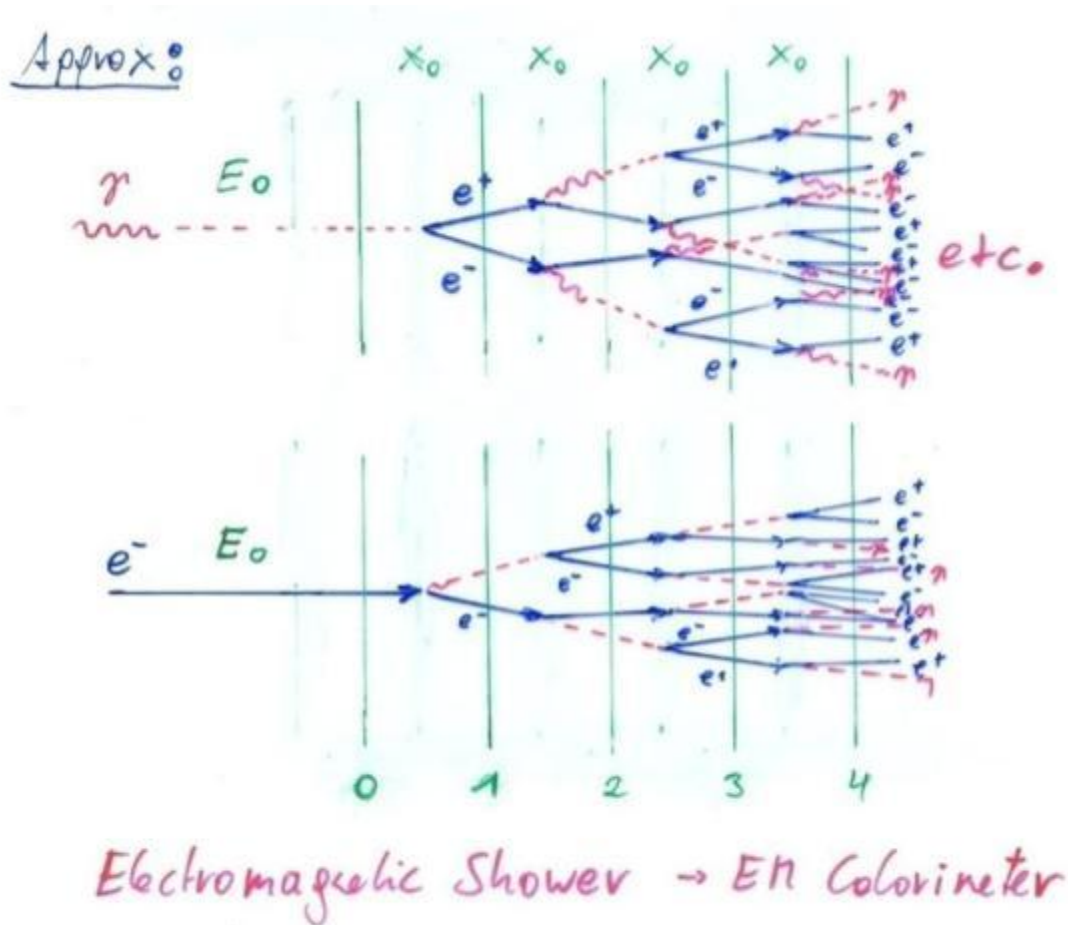
\hookrightarrow Probability that Photon converts to $e^+ e^-$ after a distance x .

For $E_\gamma \gg m_e c^2 = 0.5 \text{ MeV}$: $\lambda = 9/7 X_0$

Average distance a high energy photon has to travel before it converts into an $e^+ e^-$ pair is equal to 9/7 of the distance that a high energy electron has to travel before reducing it's energy from E_0 to $E_0 \cdot \text{Exp}(-1)$ by photon radiation.



Bremsstrahlung + Pair Production \rightarrow EM Shower

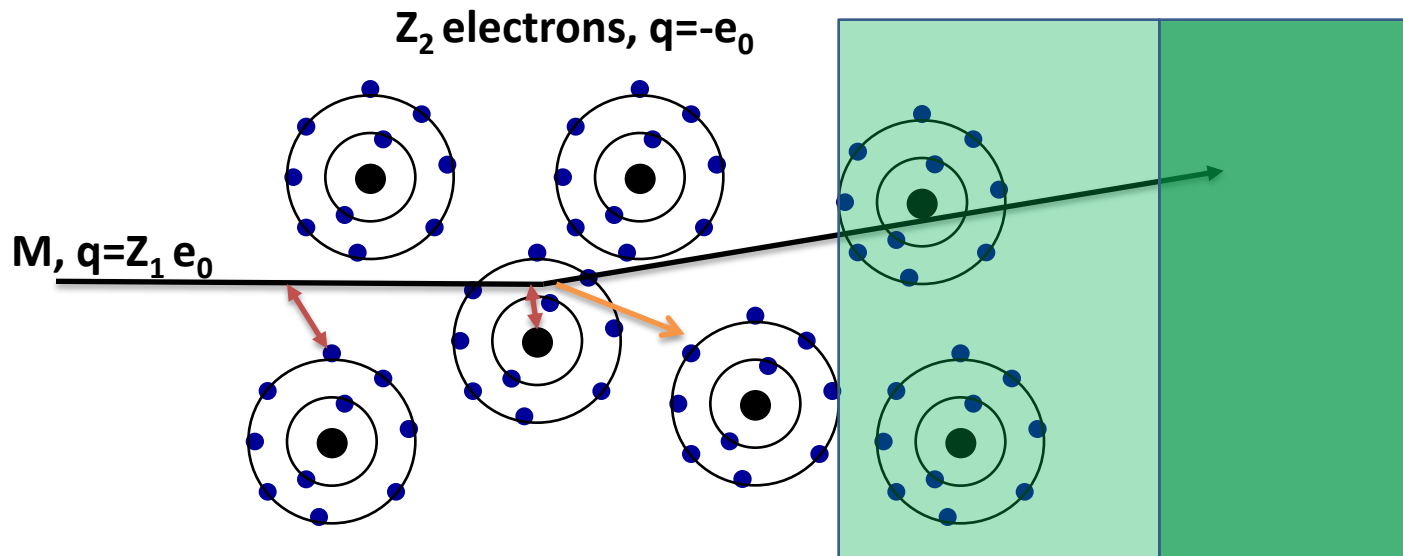


Multiple Scattering

The 'Rutherford scattering' of the incoming particle on the nuclei, that is also the reason for Bremsstrahlung, results in multiple small angle scattering scattering of the particles when traversing material.

The statistical analysis of the small angle scattering together with inclusion of the shielding effects by the electrons results in simple expressions for the multiple scattering angles of particles.

$$\frac{d\sigma}{d\Omega} = \left(\frac{1}{4\pi\epsilon_0} \frac{Z_1(Z_2 - F)e_0^2}{2pv} \right)^2 \frac{1}{\sin^4 \theta/2}$$



Multiple Scattering

Statistical (quite complex) analysis of multiple collisions gives:

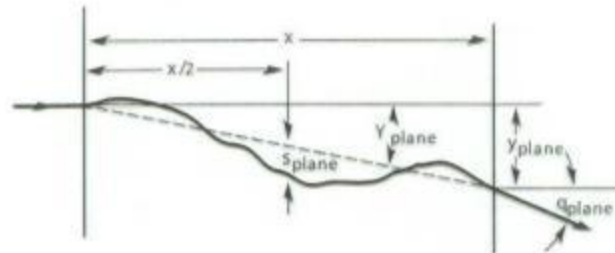
Probability that a particle is deflected by an angle θ after travelling a distance x in the material is given by a Gaussian distribution with sigma of:

$$\Theta_0 = \frac{0.0136}{\beta c p [\text{GeV}/c]} Z_1 \sqrt{\frac{x}{X_0}}$$

X_0 ... Radiation length of the material

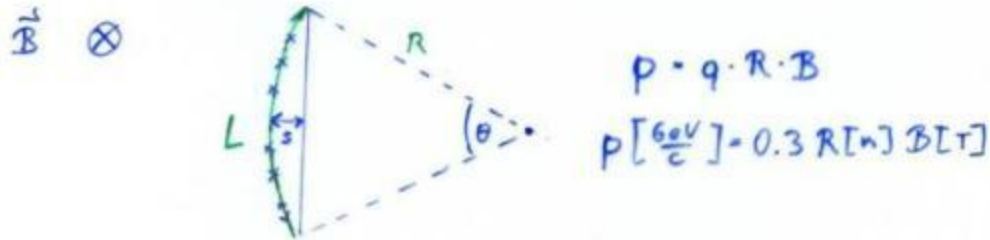
Z_1 ... Charge of the particle

p ... Momentum of the particle



Multiple Scattering

Magnetic Spectrometer: A charged particle describes a circle in a magnetic field:



$$L = R \cdot \theta$$

$$S = R \left(1 - \cos \frac{\theta}{2} \right) \sim R \frac{\theta^2}{8} = \frac{L^2}{8R} \rightarrow R = \frac{L^2}{8S}$$

$$\Delta p = 0.3 B \Delta R = 0.3 B \frac{L^2}{8S^2} \Delta S$$

$$\Delta S = \frac{\sigma_x}{\sqrt{N}} \quad \sigma_x \dots \text{point resolution, } N \dots \text{Measurement Points}$$

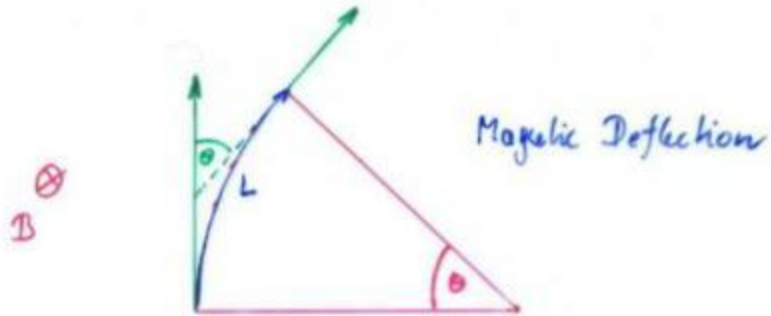
$$\frac{\Delta p}{p} = \frac{\Delta S}{S} = \frac{\sigma_x [\text{m}]}{\sqrt{N}} \cdot \frac{3.3 \cdot 8 p \left[\frac{\text{GeV}}{c} \right]}{B [\text{T}] \cdot L^2 [\text{m}^2]}$$

E.g: $p = 10 \frac{\text{GeV}}{c}$, $B = 1 \text{T}$, $L = 1 \text{m}$, $\sigma_x = 200 \mu\text{m}$, $N = 25$

$$\frac{\Delta p}{p} = 0.01 \rightarrow 1\%$$

Limit \rightarrow Multiple Scattering

Multiple Scattering



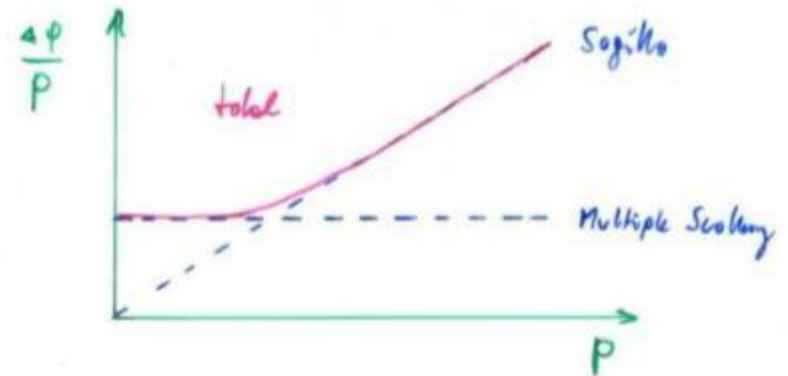
$$\rho \left[\frac{\text{GeV}}{c} \right] = 0.3 R [\text{m}] B [\text{T}]$$

$$\theta = \frac{L}{R} = \frac{L}{\rho} \cdot 0.3 B$$

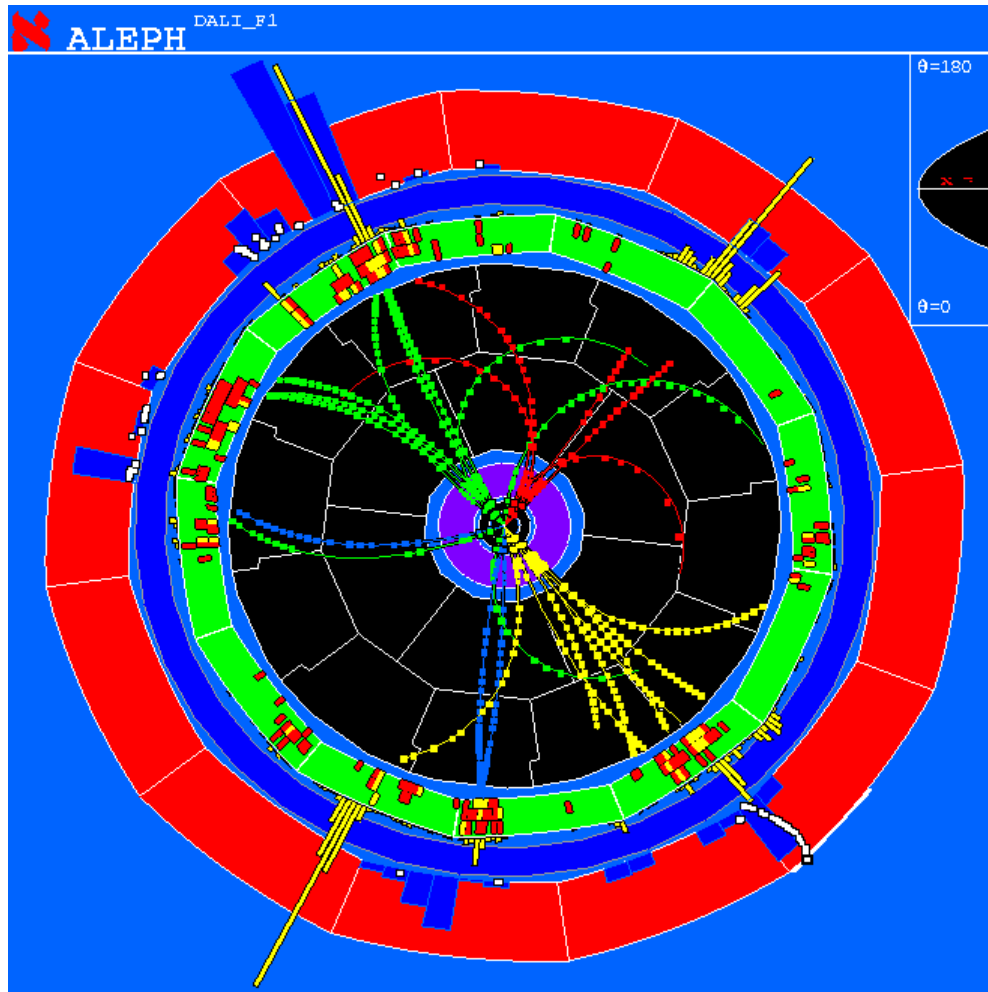
$$\frac{\Delta p}{p} = \frac{\Delta \theta}{\theta} = \frac{\theta_0}{\theta} \sim \frac{0.05}{3 B [\text{T}] L [\text{m}]} \sqrt{\frac{L}{x_0}}$$

→ Independent of p

$$\frac{\Delta p}{p} \Big|_{\text{tot}} = \sqrt{\left(\frac{\Delta p}{p} \Big|_{\text{Sog}} \right)^2 + \left(\frac{\Delta p}{p} \Big|_{\text{ms}} \right)^2}$$



Magnetic Spectrometer for measurement of Particle Momenta



Measure momentum by curvature of the particle track.

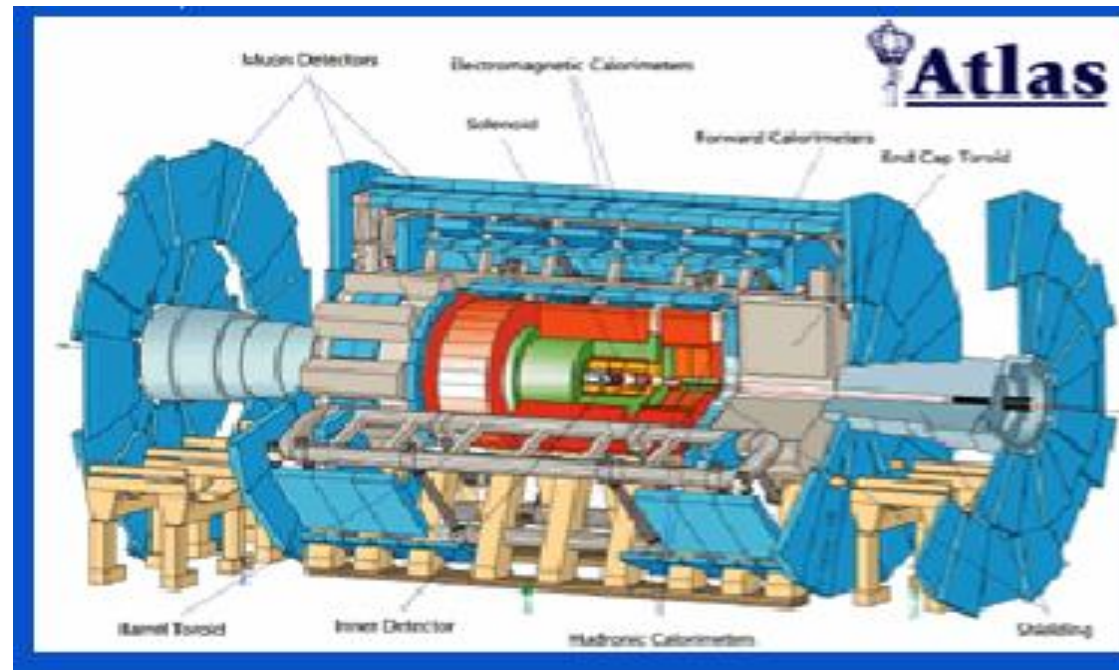
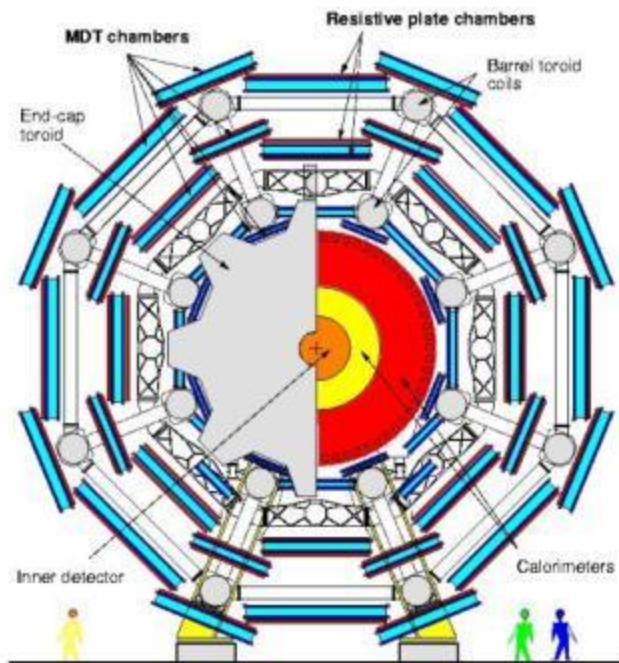
Multiple Scattering

ATLAS Muon Spectrometer:

$N=3$, $\sigma=50\mu\text{m}$, $P=1\text{TeV}$,

$L=5\text{m}$, $B=0.4\text{T}$

$\Delta p/p \sim 8\%$ for the most energetic muons at LHC



Transition Radiation

Radiation (\sim keV) emitted by ultra-relativistic particles when they traverse the border of 2 materials of different dielectric permittivity (ϵ_1, ϵ_2)



$$q = Z_1 e$$

$$I = \frac{1}{3} d Z_1^2 (\hbar \omega_p) \gamma \dots \text{Radiated Energy per Transition}$$

$\hbar \omega_p \dots$ plasma frequency of the medium
 $\dots \sim 20$ eV for Styrene

About half the Energy is radiated between

$$0.1 \hbar \omega_p \gamma < \hbar \omega < \hbar \omega_p \gamma$$

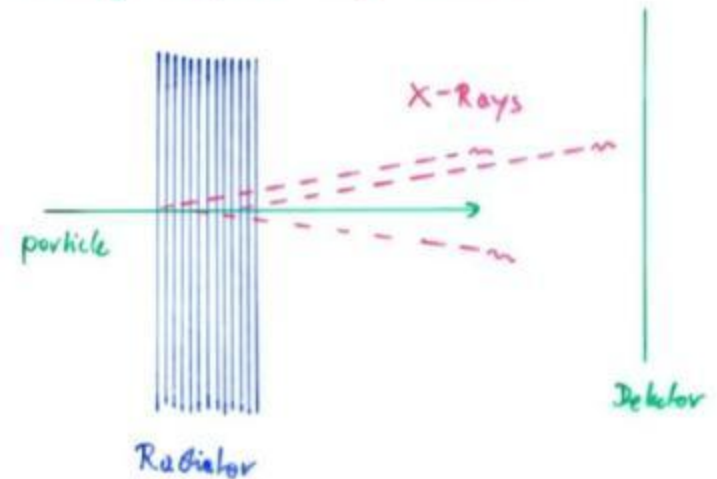
E.g. $\gamma = 1000$ 2-20 keV X-Rays

$$N_\gamma \sim \frac{2}{3} d Z_1^2 \sim 5 \cdot 10^{-3} \cdot Z_1^2$$

γ -Dependence from hardening rather than N_γ

$$\text{Emission Angle} \sim \frac{1}{\gamma}$$

The Number of Photons can be increased by placing many foils of material.



Electromagnetic Interaction of Particles with Matter

Ionization and Excitation:

Charged particles traversing material are exciting and ionizing the atoms.

The average energy loss of the incoming particle by this process is to a good approximation described by the Bethe Bloch formula.

The energy loss fluctuation is well approximated by the Landau distribution.

A more precise model of energy loss and energy loss fluctuations is the so called PAI model, which uses the measured photo absorption crosssection of the material in question for calculation of the differential energy loss crosssection for a charged particle. From this, the average energy loss and the fluctuation can be calculated very accurately.

Multiple Scattering and Bremsstrahlung:

The incoming particles are scattering off the atomic nuclei which are partially shielded by the atomic electrons.

Measuring the particle momentum by deflection of the particle trajectory in the magnetic field, this scattering imposes a lower limit on the momentum resolution of the spectrometer.

The deflection of the particle on the nucleus results in an acceleration that causes emission of Bremsstrahlungs-Photons. These photons in turn produced e^+e^- pairs in the vicinity of the nucleus, which causes an EM cascade. This effect depends on the 2^{nd} power of the particle mass, so it is only relevant for electrons.

Electromagnetic Interaction of Particles with Matter

Cherenkov Radiation:

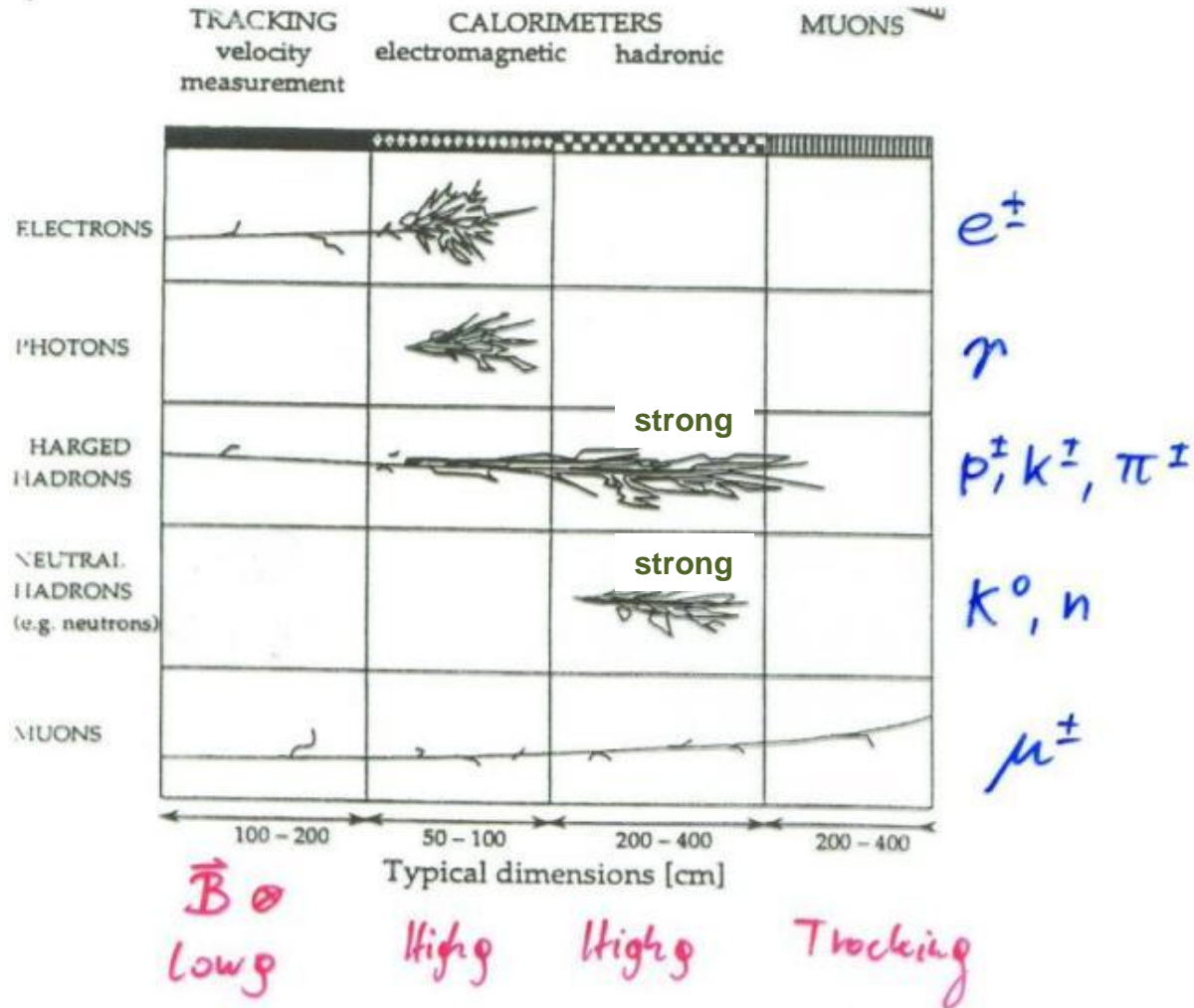
If a particle propagates in a material with a velocity larger than the speed of light in this material, Cherenkov radiation is emitted at a characteristic angle that depends on the particle velocity and the refractive index of the material.

Transition Radiation:

If a charged particle is crossing the boundary between two materials of different dielectric permittivity, there is a certain probability for emission of an X-ray photon.

→ The strong interaction of an incoming particle with matter is a process which is important for Hadron calorimetry and will be discussed later.

Electron-Magnetic Interactions with the Detectors



Messung von Photonen

- 1) UV photonen: $E > 5\text{eV}$
- 2) Röntgenstrahlen: $1\text{keV} < E < 1\text{MeV}$
- 3) Gamma Strahlen: $E > 1\text{MeV}$

Fällt ein Photon auf Materie ein und ist der Atomare Streuquerschnitt für eine Interaktion des Photons $= \sigma$, so ist die Wahrscheinlichkeit dass das Photon bei einer zwischen Distanz x und $x+dx$ eine Wechselwirkung erfährt

$$P(x) = 1/\lambda \text{ Exp}(-x/\lambda)$$

wobei $\lambda = A/N_A \rho \sigma$ die mittlere freie Weglänge ist. A und ρ sind Atomare Massenzahl und Dichte des Materials.

Fällt ein (monochromatischer) Photonenstrahl der Intensität I_0 (photonen/s) auf das Material, so ist die Intensität $I(x)$ des Strahls nach Distanz x im Material

$$I(x) = I_0 \text{ Exp}(-x/\lambda) = I_0 \text{ Exp}(-\mu x)$$

$\mu = N_A \rho \sigma / A$ wird als Massenabschwächungskoeffizient bezeichnet.
Für mehrere Wechselwirkungsprozesse ist

$$\mu = N_A \rho / A \sum \sigma_i$$

Messung von Photonen

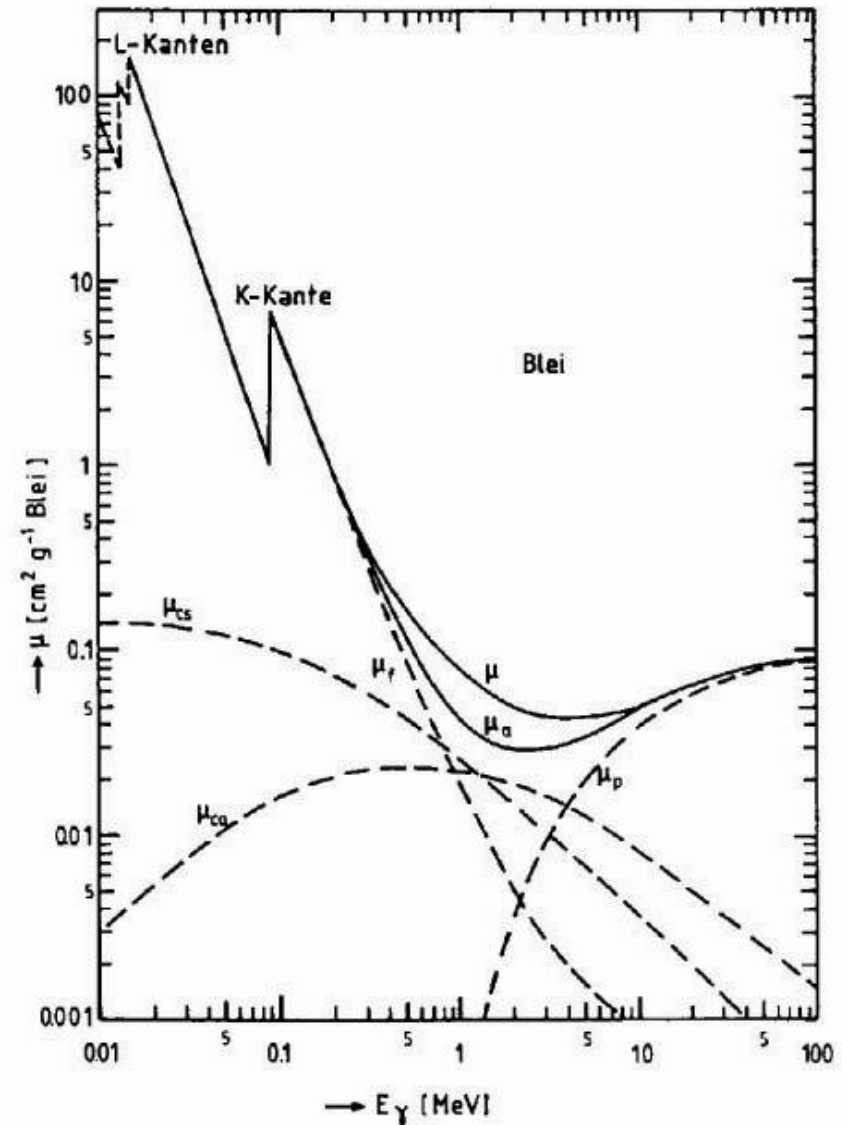
Nachweisprinzip:

Wechselwirkung der Photonen im
Detektormaterial

Photoionisation
Comptonstreuung
Paarproduktion

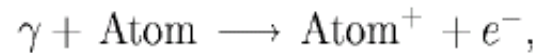
Die dabei erzeugten Elektronen und
Positronen werden im Detektorgas (dE/dx Energie-Verlust) absorbiert. Die dabei erzeugte Ionisation wird gemessen.

μ/ρ



Messung von Photonen

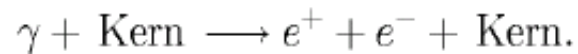
Für kleine Energie (Ionisations $E \leq E_\gamma \leq 100 \text{ keV}$) dominiert der *Photoeffekt*



für Energie $E_\gamma \sim 1 \text{ MeV}$ der *Compton-Effekt*



und bei hohen Energien ($E_\gamma \gg 1 \text{ MeV}$) die *Paarerzeugung*



Photoeffekt:

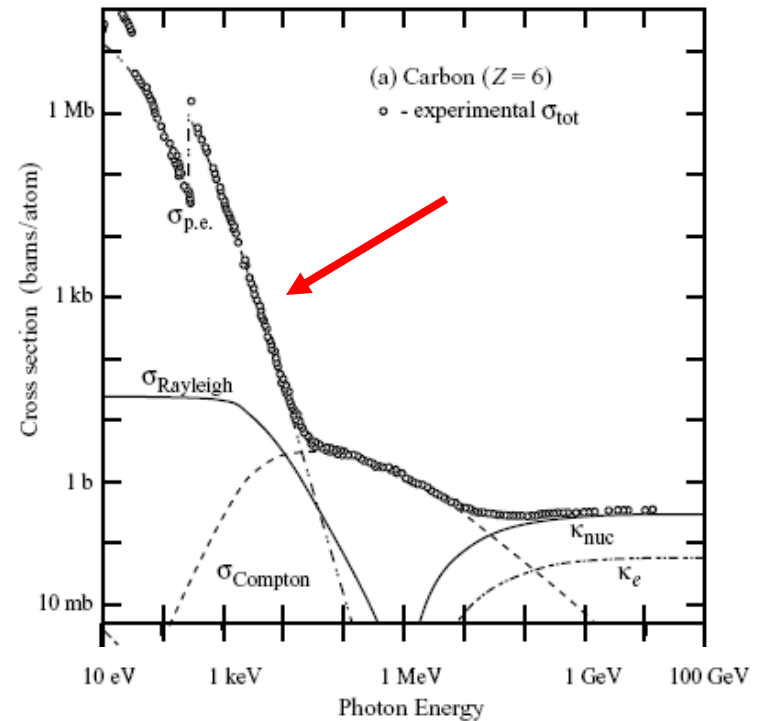
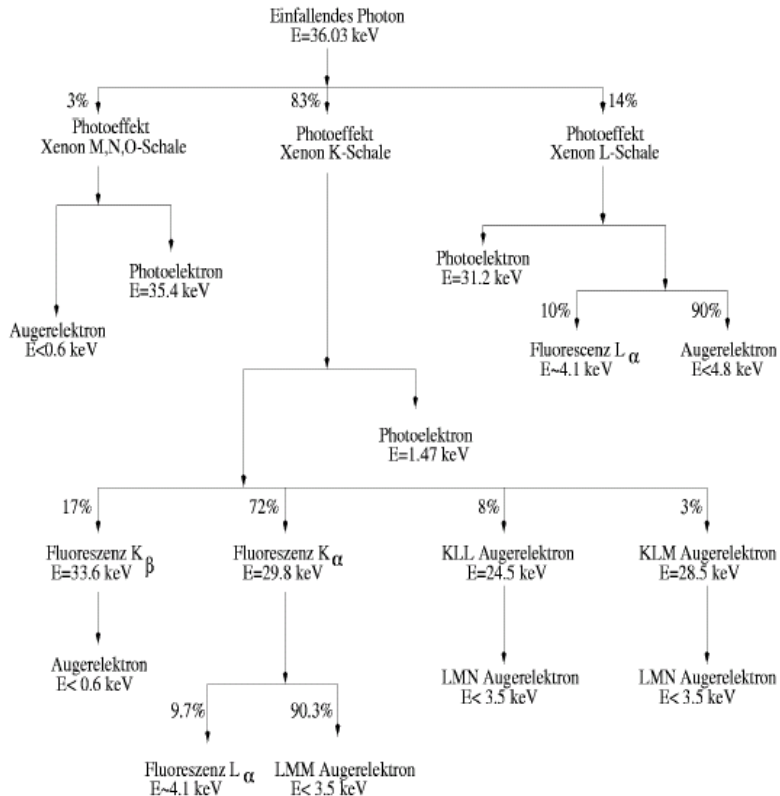
$$\sigma_{Ph} = \left(\frac{32}{\epsilon^7}\right)^{\frac{1}{2}} \alpha^4 Z^5 \sigma_{Th}^2$$

$$\epsilon = \frac{E_\gamma}{m_e c^2} \quad \sigma_{Th} = \frac{8}{3} \pi r_e^2$$

Für hohe Energie ($\epsilon \gg 1$) gilt:

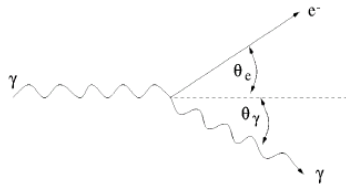
$$\sigma_{Ph} = 4\pi r_e^2 Z^5 \alpha^4 \frac{1}{\epsilon}$$

Als Folge des Photoeffekts in einer inneren Schale treten folgende Sekundäreffekte auf:



Compton Effekt:

Streuung eines Photons an einem Elektron



$$\frac{E_{\gamma'}}{E_{\gamma}} = \frac{1}{1 + \varepsilon(1 - \cos \theta)}$$

Differentieller Wirkungsquerschnitt (Klein-Nishina):

$$\frac{d\sigma}{d\Omega} = \frac{r_e^2}{2} \frac{1}{[1 + \varepsilon(1 - \cos \theta)]^2} \left(1 + \cos^2 \theta + \frac{\varepsilon^2(1 - \cos \theta)^2}{1 + \varepsilon(1 - \cos \theta)} \right)$$

Totaler Wirkungsquerschnitt:

$$\sigma_c^e = 2\pi r_e^2 \left[\left(\frac{1 + \varepsilon}{\varepsilon^2} \right) \left\{ \frac{2(1 + \varepsilon)}{1 + 2\varepsilon} - \frac{1}{\varepsilon} \ln(1 + 2\varepsilon) \right\} + \frac{1}{2\varepsilon} \ln(1 + 2\varepsilon) - \frac{1 + 3\varepsilon}{(1 + 2\varepsilon)^2} \right]$$

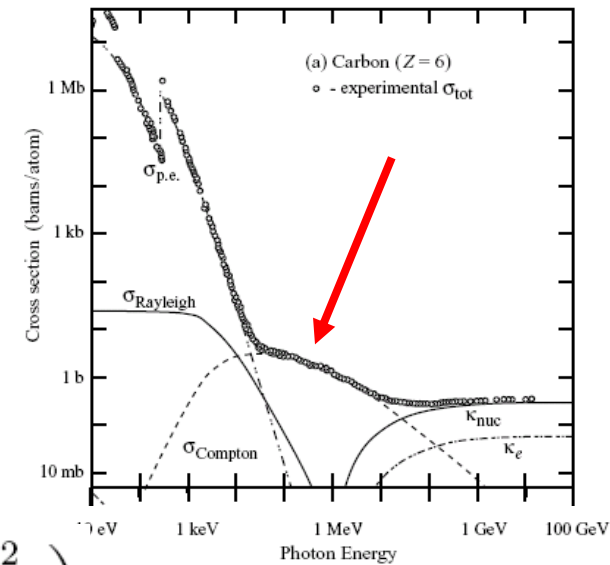
Differentieller Absorptionsquerschnitt:

$$\frac{d\sigma_{ca}}{d\Omega} = \frac{E_{\gamma} - E_{\gamma'}}{E_{\gamma}} \frac{d\sigma}{d\Omega}$$

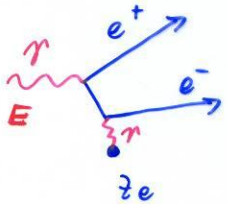
Totaler Absorptionsquerschnitt:

$$\sigma_{ca} = \sigma_c^e - \pi r_e^2 \left[\frac{1}{\varepsilon^3} \ln(1 + 2\varepsilon) + \frac{2(1 + \varepsilon)(2\varepsilon^2 - 2\varepsilon - 1)}{\varepsilon^2(1 + 2\varepsilon)^2} + \frac{8\varepsilon^2}{3(1 + 2\varepsilon)^3} \right]$$

Atomarer Absorptionsquerschnitt: $\sigma = Z^* \sigma_{ca}$

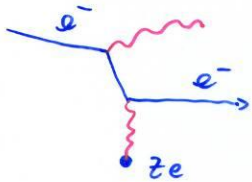


High Energy γ Interaction: Pair Production



$$\gamma + Nucl. \rightarrow e^+ + e^- + Nucl.$$

The Diagram is very similar to Bremsstrahlung



$e^- + Nucl. \rightarrow \gamma + e^- + Nucl.$
 "Crossing Symmetry: bring particle to the other side and make it the antiparticle" \rightarrow "same" correction ...

$$\frac{d\sigma(E, E')}{dE'} = 4\alpha Z^2 v_e^2 \frac{1}{E} \cdot G(E, E') \quad E \gg 137 m_e c^2 Z^{-1/3}$$

$$G(E, E') = \left[\left(\frac{E'+m_e c^2}{E} \right)^2 \left(1 - \frac{E'+m_e c^2}{E} \right)^2 + \frac{2}{3} \frac{E'+m_e c^2}{E} \left(1 - \frac{E'+m_e c^2}{E} \right) \ln \frac{E}{E'} - \frac{1}{3} \frac{E'+m_e c^2}{E} \left(1 - \frac{E'+m_e c^2}{E} \right) \right]$$

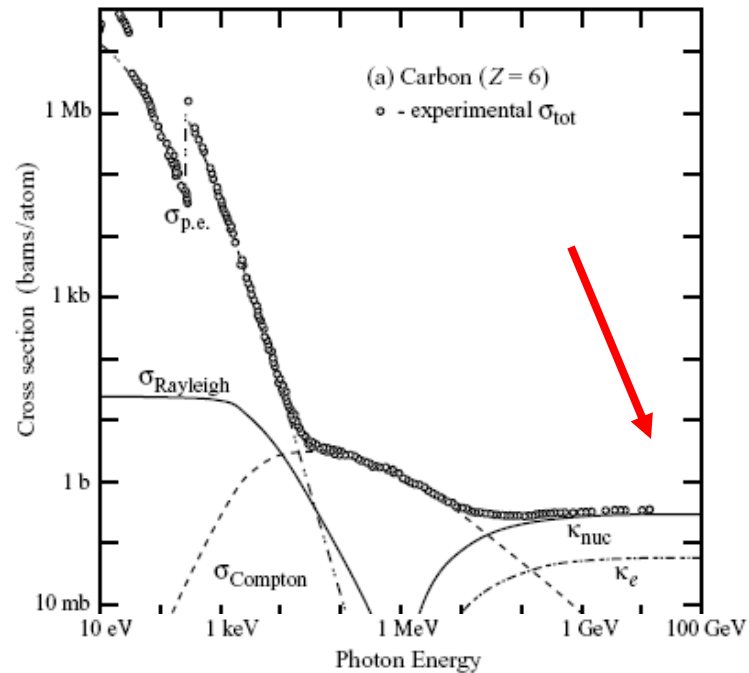
$$\sigma = \int_0^{E-2m_e c^2} \frac{d\sigma}{dE'} dE' = 4\alpha Z^2 v_e^2 \cdot \frac{7}{9} \ln 183 Z^{-1/3}$$

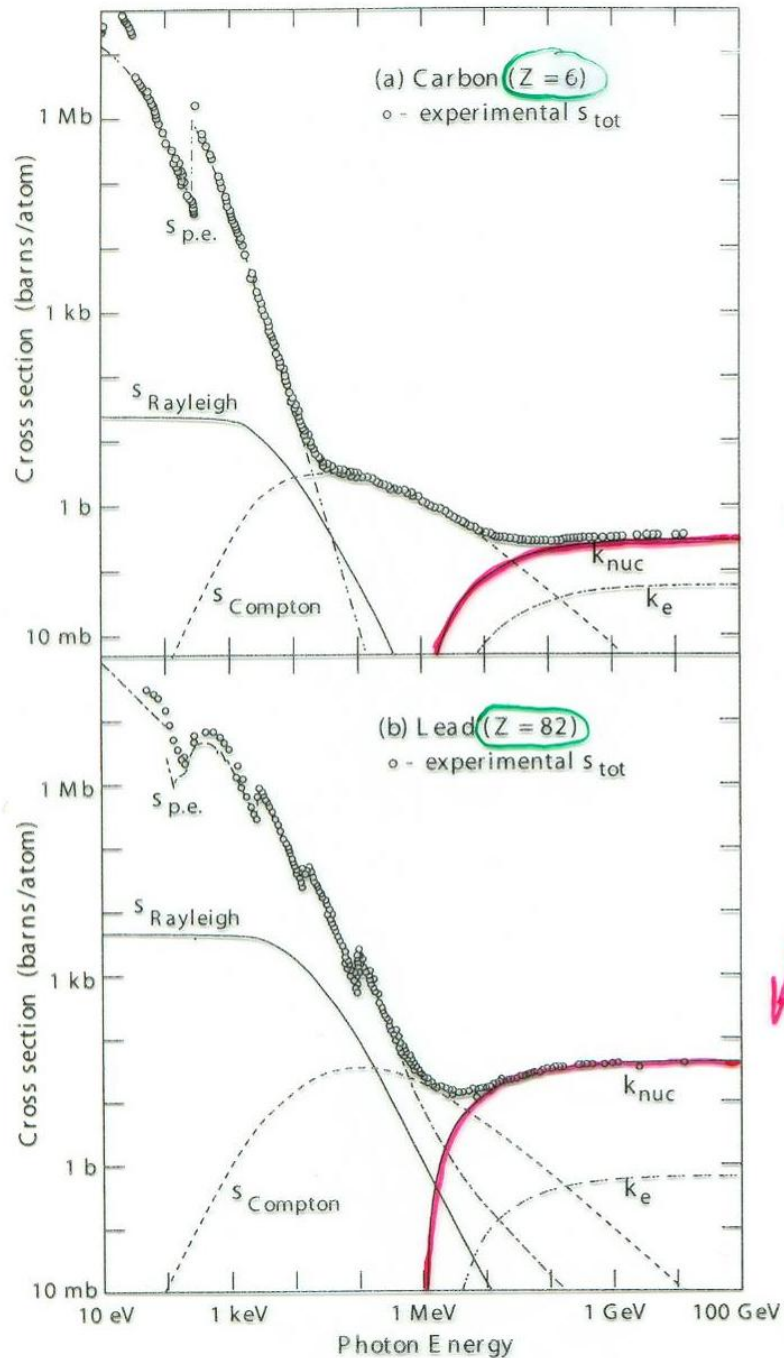
$$P(x) = \frac{1}{\lambda} e^{-\frac{x}{\lambda}} \quad \lambda = \frac{A}{9 N_A \sigma} = \frac{9}{7} X_0$$

\hookrightarrow Probability that Photon converts to $e^+ e^-$ after a distance x .

Für $E_\gamma \gg m_e c^2$: $\lambda = 9/7 X_0$

Mittlere Distanz welche ein Photon zurücklegt bevor es ein $e^+ e^-$ Paar bildet ist gleich 9/7 der Distanz welche ein hochenergetisches Elektron der Energie E_0 zurücklegt bevor seine Energie auf Grund von Bremsstrahlung auf $E_0 \cdot \text{Exp}(-1)$ gefallen ist.





Pair Production

$$E_\gamma > 2m_e c^2 \sim 1 \text{ MeV}$$

$$\left(\frac{82}{6}\right)^2 \sim 187$$



Nachweis von Photonen in Gasdetektoren

Nachweisprinzip:

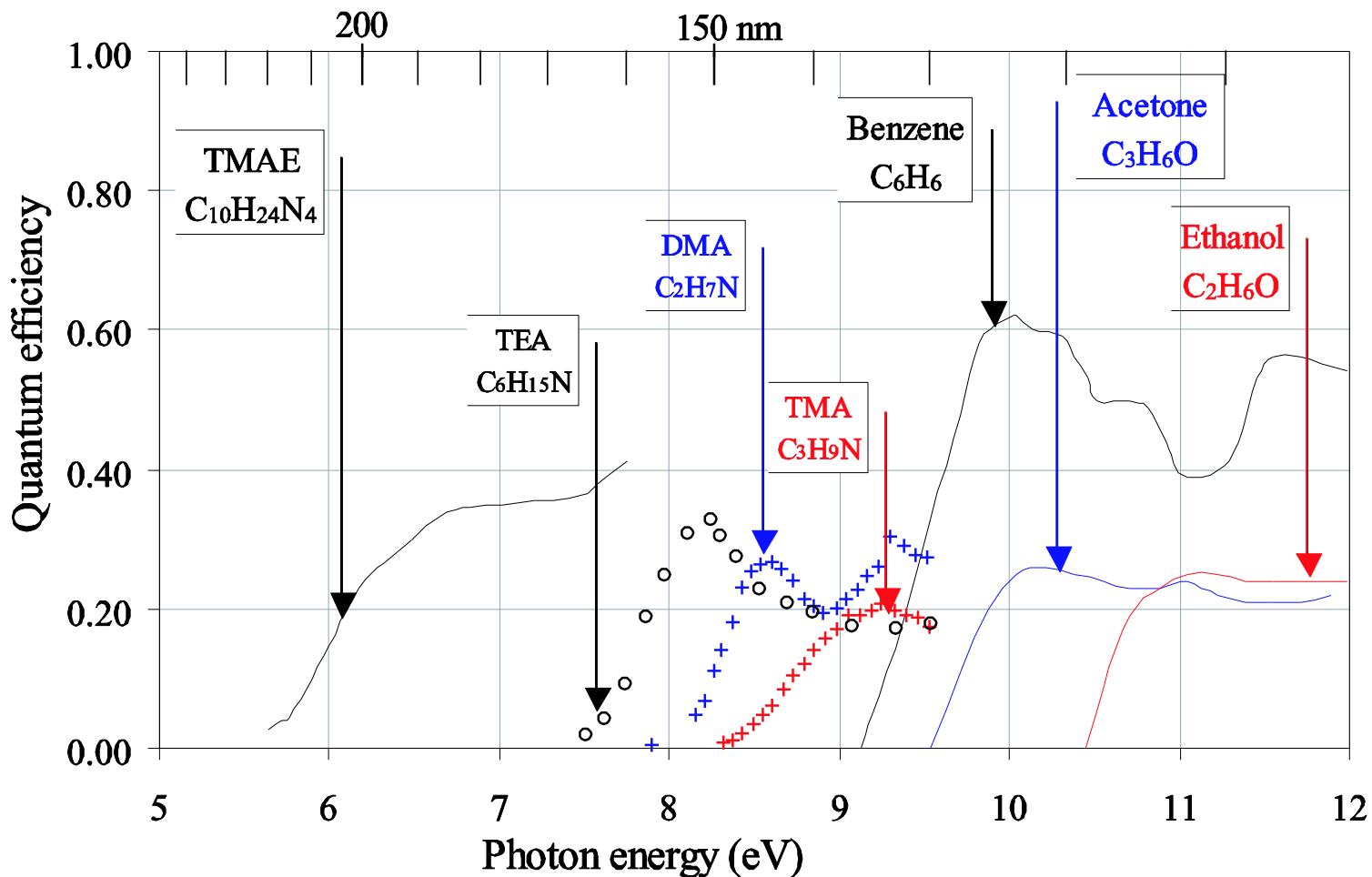
Wechselwirkung der Photonen im
Detektormaterial

Photoionisation
Comptonstreuung
Paarproduktion

Die dabei erzeugten Elektronen und
Positronen werden im Detektorgas (dE/dx Energie
Verlust) absorbiert. Die dabei erzeugte Ionisation wird
gemessen.

Nachweis von UV-Photonen durch Ionisation organischer Dämpfe

Entscheidend: Transmission der UV-Photonen im Detektor-Gas (Ort der Emission)
Absorptionswahrscheinlichkeit im Photonendetektor ('Quanteneffizienz')



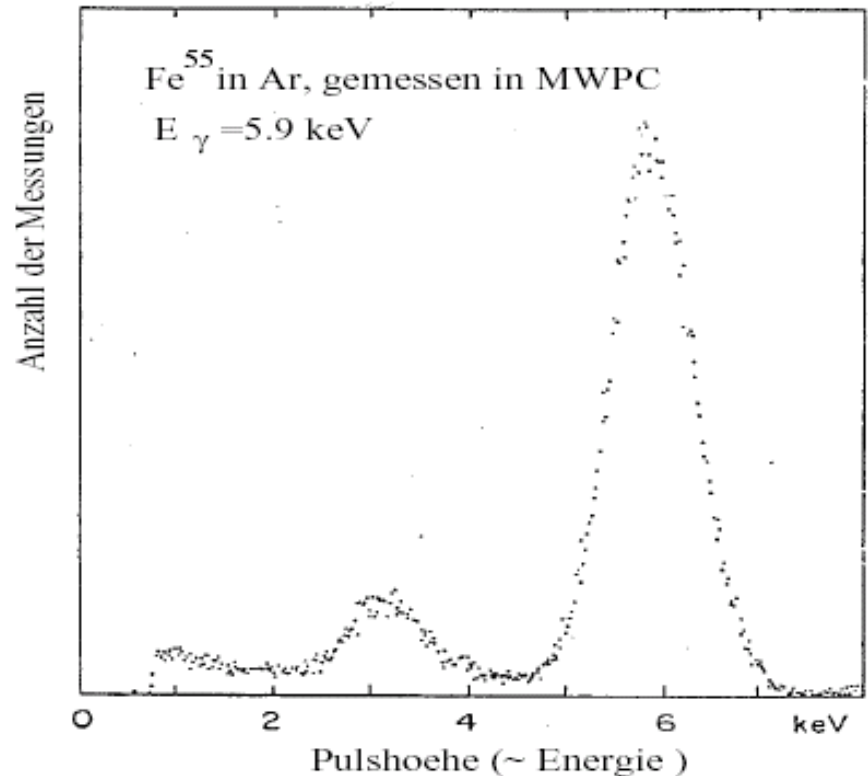
Beispiel: Absorption eines 5.9 keV Röntgenphotons in Argon

~96% werden an der K-Schale
($E=3.2\text{keV}$) absorbiert →
Energie des Photoelektrons: $5.9-3.2=2.7\text{keV}$

80%: Vakanz wird durch einen 'Auger'-
Übergang gefüllt. Dabei wird ein Elektron
mit der K-Schalen-Energie von 3.2keV
emittiert. Abgegebene Gesamtenergie:
 $2.7+3.2\text{keV} = 5.9\text{keV}$

16%: M-L Fluoreszenz mit Emission
eines Photoelektrons aus der L-Schale:
Energie= 0.3keV . Abgegebene
Gesamtenergie: $2.7+0.3\text{keV} = 3\text{keV}$

~4% werden an L-Schale (Energie= 0.3keV)
absorbiert; Photoelektron $5.9-0.3=5.6\text{keV}$,
gefolgt von M-L Fluoreszenz
und 0.3keV Photoelektron



Energieauflösung $\sigma/E \approx 1/\sqrt{N}$
 N ...Anzahl der Elektronen = E_γ/I
Argon $E_\gamma=5.9\text{keV}$, $I=26\text{eV}$, $N=225$
 $\sigma/E \approx 6.6\%$.

Röntgenstrahlungsmessung: Anwendungen

- **5-30 keV Bereich**
 - **Strukturanalyse (z.B: Proteine) : im Vergleich zur Filmregistrierung 100 bis 1000 mal höhere Empfindlichkeit, führte zu zahlreichen optimierten Detektorentwicklungen**
 - **Medizinische Diagnostik**
 - **‘Übergangsstrahlungsdetektoren’ → Teilchenidentifizierung**
- **30keV bis 1 MeV**
 - **Kernmedizin: Positron-Emission-Tomograph (PET): Positronemitter injiziert, der sich präferenziell in Tumoren anlagert: Emission von zwei γ -Quanten mit je 511keV Energie: Lokalisierung der Emission ermöglicht Lokalisierung des Tumors. Lokalisierung des Photons mittels Konvertern (z.B. Blei)**
 - **Materialprüfung: Analyse von Materialien mit γ -Quellen**

Strukturanalyse, Materialprüfung, Sicherheitsanlagen mit Drahtkammermethoden

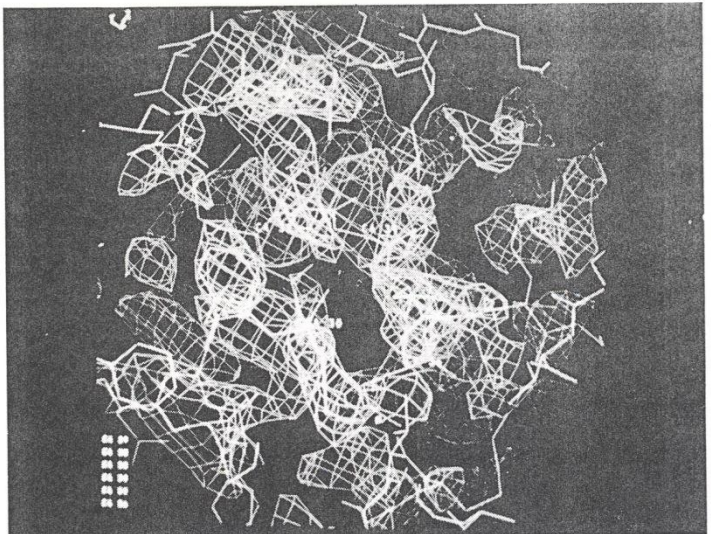
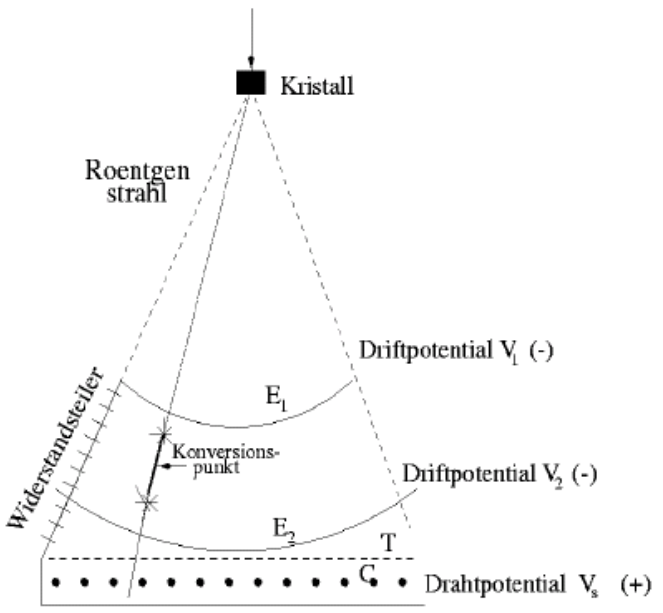
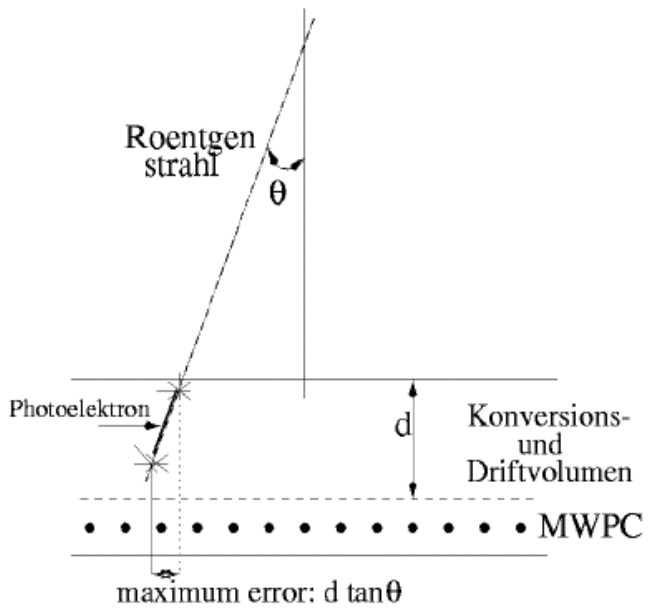


Fig. 29 Example of the electronic structure of a macromolecule reconstructed at Orsay using diffraction measurements carried out with the spherical chamber shown in the previous figure. (Photo LAL, Orsay)

(Photo LAL, Orsay)

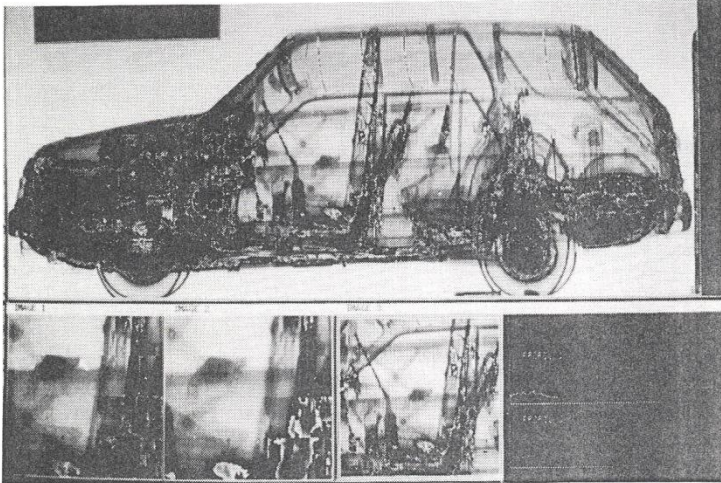


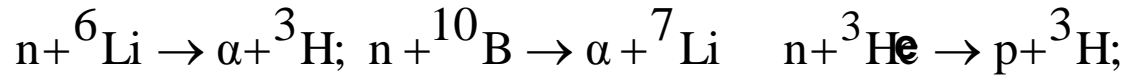
Fig. 30 Radiograph of a car made by the method based on the use of a multiwire proportional chamber with a thin converter and a grazing incidence of the hard-X-ray beam. (Photo Schlumberger)

Nachweis von Neutronen

Nachweisprinzip:

Starke Wechselwirkung mit dem aktiven Detektormaterial. In Abhängigkeit von der Neutronenergie – verschiedene Kernreaktionen

$E < 20 \text{ MeV}$: Nachweis geladener Sekundärteilchen



$E < 1 \text{ GeV}$: Messung der Rückstossprotonen

$$E_p^\Theta = \frac{4M_n M_p}{(M_n + M_p)^2} E_n \cos^2 \Theta \quad E_p^{\max} \approx E_n$$

$E > 1 \text{ GeV}$: Messung der Rückstossprotonen

Messung der durch starke Wechselwirkung erzeugten Sekundärteilchen (hadronische Schauer \rightarrow Kalorimetrie)

Nachweis von Neutronen

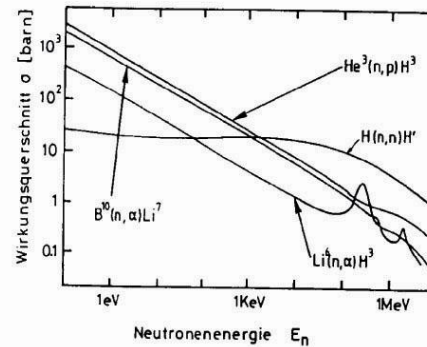


Abbildung 3.9: Wirkungsquerschnitte fuer neutroneninduzierte Kernreaktionen als Funktion der Neutronenenergie E_n .

Höherenergetische Neutronen müssen erst ‘moderiert’ werden (durch inelastische Stösse), um sie dann in einem Detektor nachweisen zu können

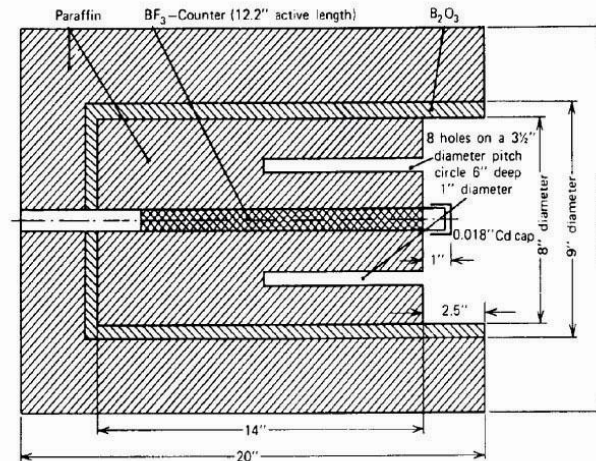
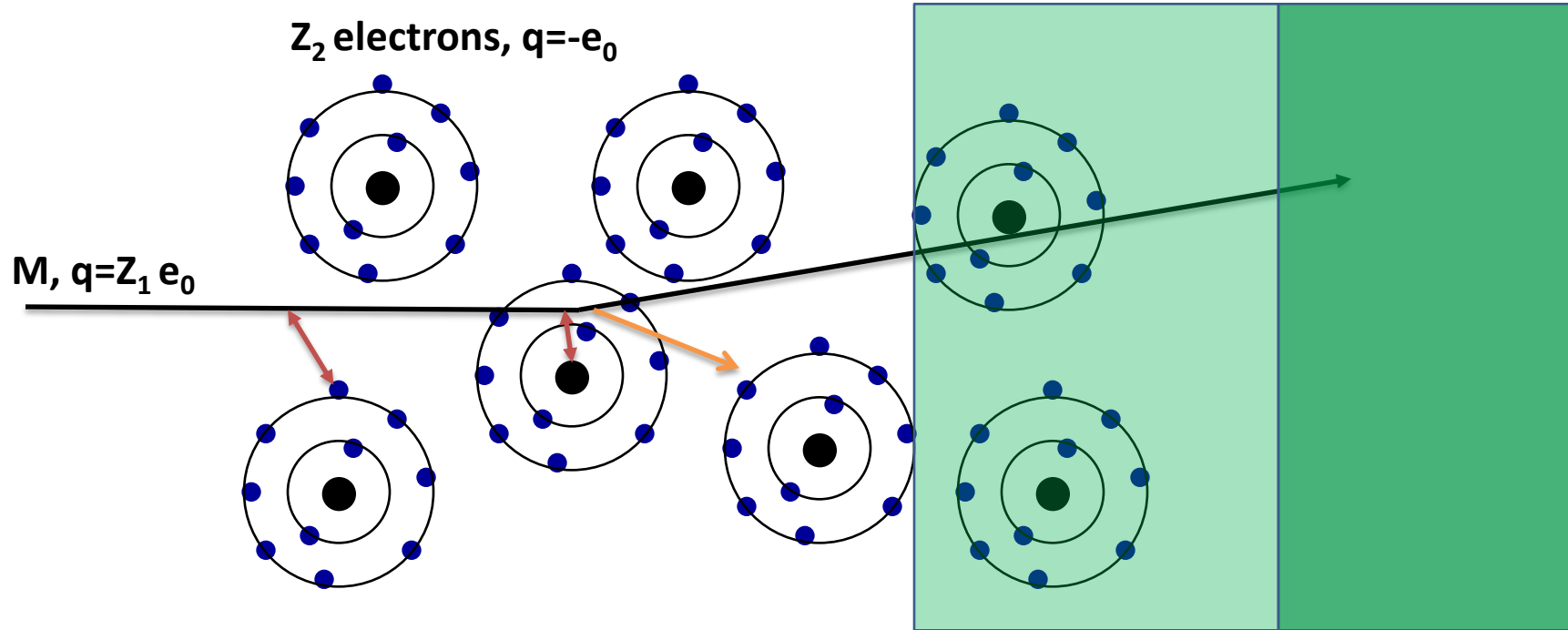


Abbildung 3.10: Die Neutronen werden durch elastische Streuung im Paraffin moderiert und anschliessend im BF_3 Zählrohr gemessen.

Electromagnetic Interaction of Particles with Matter



Now that we know all the Interactions we can talk about Detectors !

Interaction with the atomic electrons. The incoming particle loses energy and the atoms are excited or ionized.

Interaction with the atomic nucleus. The particle is deflected (scattered) causing multiple scattering of the particle in the material. During this scattering a Bremsstrahlung photon can be emitted.

In case the particle's velocity is larger than the velocity of light in the medium, the resulting EM shockwave manifests itself as Cherenkov Radiation. When the particle crosses the boundary between two media, there is a probability of the order of 1% to produced and X ray photon, called Transition radiation.